Version RSL/4

EGT3
ENGINEERING TRIPOS PART IIB

Wednesday 25 April 2018 2 to 3.40

Module 4C7

RANDOM AND NON-LINEAR VIBRATIONS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C7 Random and Non-linear Vibrations data sheet (4 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A single degree of freedom system representing an energy harvesting device is shown in Fig. 1. The system has mass M, spring stiffness K, and damper rate C, and the excitation arises from random base displacements $y(t)$ and an applied random force $F(t)$. The displacement of the mass of the device is $x(t)$ as shown in the figure, and the relative displacement between the mass and the base is denoted by $r(t) = x(t) - y(t)$.

(a) For the case where $F(t) = 0$ and the base acceleration $\ddot{y}(t)$ is white noise with double sided spectral density S_{AA}, show that the average rate of power dissipation in the damper is independent of both the damping and stiffness of the device and proportional to the mass. [25%]

(b) For an alternative case where $y(t) = 0$ and $F(t)$ is white noise with double sided spectral density S_{FF}, show that the average power dissipated by the damper is again independent of the damping and stiffness of the device, but in this case inversely proportional to the mass. [15%]

(c) If the base motion and the force are applied simultaneously, and they are statistically independent, find an expression for the system mass that minimises the average power dissipated by the damper. [20%]

(d) For the case described in part (a), find expressions for the mean squared values of $r(t)$ and $x(t)$. In the light of your result for $x(t)$, comment on the conditions that must be met for white noise to be a reasonable approximation for the acceleration input. [20%]

(e) For the case described in part (b), explain how you would calculate the average power dissipated were $F(t)$ not white noise. [20%]

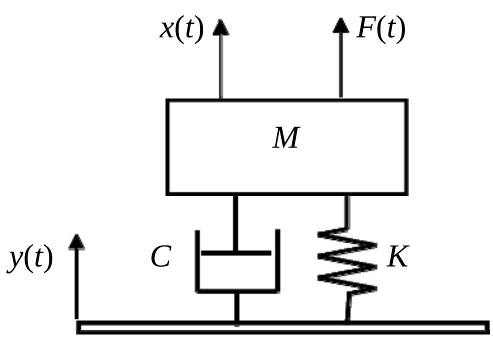

Fig. 1

2 (a) Define the terms *stationary* and *ergodic* in the context of a stochastic process, and give an example of a process that is stationary but not ergodic. [20%]

(b) Explain how, under certain conditions, it is possible to derive the joint probability density function of a stochastic process and its velocity from knowledge of the spectrum of the process. Give one example of a situation in which these conditions are *not* met. [15%]

(c) The standard analytical result for the crossing rate of a random process is valid regardless of the bandwidth of the random process. Explain why this is not true for the standard approximations to (i) the probability of exceeding a particular level, and (ii) the probability density function of peak heights. [20%]

(d) The underside of a North Sea offshore oil production platform is 18 m above the mean level of the sea. In a severe January storm, the *single-sided* spectrum of the sea surface elevation is measured to be

$$S(\omega) = 225\omega \exp(-8\omega^2) \quad \text{m}^2\text{s/rad}$$

Estimate the mean peak height of the waves, and the probability that a wave will impact the underside of the deck at least once during a storm of duration three hours. Do you think the design of the platform is acceptable? [45%]

3 An undamped non-linear system has a symmetrical force-displacement relationship as shown in Fig. 2 below.

(a) Sketch the input and output waveforms when the system is sinusoidally driven with a constant amplitude α for each of the three cases $\alpha > \gamma$, $\gamma > \alpha > \beta$, and $\alpha < \beta$. [30%]

(b) (i) Derive an expression for the Describing Function of this system for input amplitudes $\alpha > \gamma$, $\gamma > \alpha > \beta$, and $\alpha < \beta$. [30%]

(ii) Show that the Describing Function of this system reduces to the case of a switching controller as $\beta \to 0$ and $\gamma \to \infty$, as would be expected. [20%]

(c) If the system is driven by a sinusoidal force $f \cos \omega t$, determine an approximate relationship between the response amplitude α and the frequency ω, considering the case $\alpha > \gamma$. [20%]

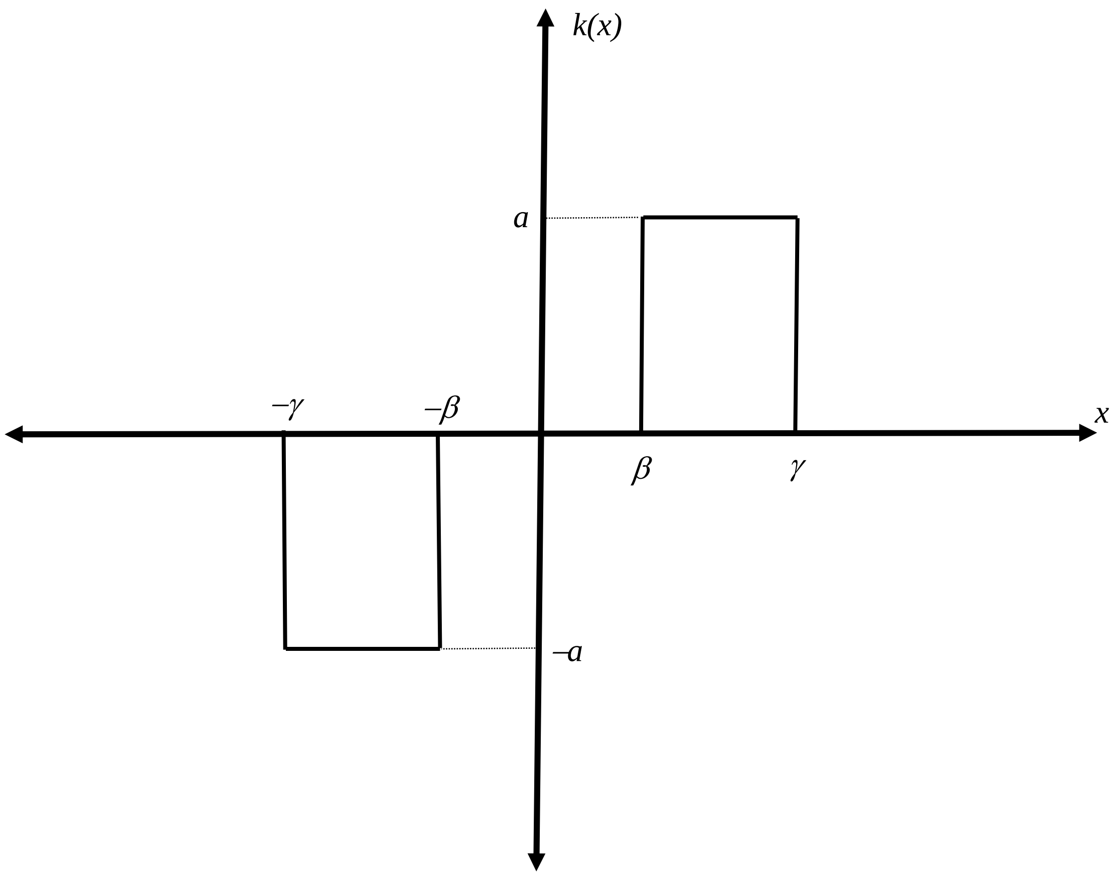

Fig. 2

4 The motion of a conservative system is governed by the following equation

$$\ddot{u} + u = \varepsilon u^2 ,$$

where ε is a small positive real parameter.

(a) Identify the equilibrium or singular points of the system. [20%]

(b) Determine the nature of the singular points. [20%]

(c) Sketch the behaviour of the system in the phase plane. [30%]

(d) The initial conditions for the system are given by $u = A$ and $\dot{u} = 0$ at $t = 0$. Use the method of iteration to obtain a solution for the response to first order in ε. [30%]

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ANSWERS

1. (c) $M = (S_{FF} / S_{AA})^{1/2}$, (d) σ_x^2 is infinite.

2. (d) P=0.006.

3. (b) For $\alpha > \gamma$ for example $D = (4a / \pi\alpha)\left\{\sqrt{1-(\beta / \alpha)^2} - \sqrt{1-(\gamma / \alpha)^2}\right\}$

4. (b) Centre and saddle point.

   (d) $u = (A - \varepsilon A^2 / 3)\cos t + (\varepsilon A^2 / 2) - (\varepsilon A^2 / 6)\cos 2t$