Version RSL/3

EGT3
ENGINEERING TRIPOS PART IIB

Thursday 25 April 2019 14.00 to 15.40

Module 4C7

RANDOM AND NON-LINEAR VIBRATIONS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C7 Random and Non-linear Vibrations data sheet (4 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A vehicle is travelling along a rough road at speed *V*. The surface elevation of the road can be represented as a Gaussian random process $u(x)$, where x is the distance along the road, and the autocorrelation function of the elevation is given by

$$\mathrm{E}[u(x)u(x+X)] = A\mathrm{e}^{-\alpha|X|}$$

where A and α are constants. The dynamics of the vehicle are represented by a single degree of freedom simplified "quarter-car" model, as shown in Fig. 1, with mass M, suspension stiffness K, and suspension damping C. The wheel is in contact with the ground and moves up and down due to the road roughness.

(a) Derive an expression for the displacement of the wheel as a function of time, and hence find the autocorrelation function of the displacement as a function of the time lag τ. Find the spectrum of the wheel displacement. [25%]

(b) Find the equation of motion of the system and hence derive an expression for the spectrum of the force in the suspension spring, based on the *relative* motion between the vehicle and the ground. [30%]

(c) By assuming that the spectrum of the wheel motion is broad banded relative to the frequency response function of the system, derive an repression for the mean squared force in the suspension spring. [25%]

(d) If the vehicle is very heavy (consider the case $M \to \infty$) show that the *exact* answer for the mean squared force in the spring is $K^2 A$. Explain why your approximate answer in part (c) does not predict this result. [20%]

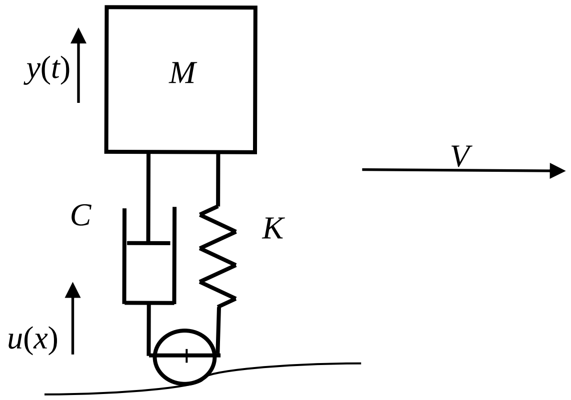

Fig. 1

2 Two circuit boards are mounted in close proximity in a rack in a satellite structure. There is a concern that the boards may impact each other due to the random vibration that occurs during launch. If impact occurs, it will occur between the central points of each board; the relevant motion of the centre of the first board is denoted by $x(t)$ and the motion of the centre of the second board is denoted by $y(t)$, so that the relative motion is $x(t) - y(t)$. The rack is designed so that in the absence of vibration the boards are separated by a distance d = 5 mm. The most severe vibration occurs over a three minute interval, during which time the vibration levels are

$$\sigma_x = 0.6 \text{ mm}, \quad \sigma_y = 0.8 \text{ mm}, \quad \sigma_{\dot{x}} = 122 \text{ mm/s}, \quad \sigma_{\dot{y}} = 274 \text{ mm/s}.$$

The motion of the boards is Gaussian and uncorrelated.

(a) Calculate the probability that the boards will impact each other during launch. [30%]

(b) To reduce the probability of impact, the decision is made to add damping to each of the boards. Assuming that the excitation acting on the system is very broad-banded, calculate the new probability of failure if the added damping has the effect of doubling the damping coefficient of each of the boards. Do you think the new probability is acceptable? [35%]

(c) There is a concern that the first board may suffer severe fatigue damage during launch. The stress in the board is found to be $S(t) = 0.3x(t)$ N/mm^2 and the S-N law for fatigue damage of the material is

$$N(S) = 4000 S^{-1}.$$

Calculate the fatigue damage that occurs when the system has the original level of damping. Is this an acceptable level of damage? [35%]

(TURN OVER

3 A single degree of freedom system has the nonlinear equation of motion

$$\ddot{x} - \varepsilon \dot{x}(3 - 8x^4) + x = 0$$

where ε is a positive constant with $\varepsilon \ll 1$ so that the middle term in the equation is small.

(a) Show that the origin of the phase plane (0,0) is an unstable focus. [20%]

(b) Use the method of iteration to solve for the response of the system to first order, assuming that the response converges to a stable limit cycle. [50%]

(c) Sketch the behaviour of the system in the phase plane. [30%]

4 Consider the Duffing system defined by the equation

$$\ddot{x} + p^2 x + \mu x^3 = K \cos \omega t$$

(a) Use the Describing Function approach to obtain a solution of the form $x_0(t) = a \cos \omega t$ and determine the frequency response function for this case. [30%]

(b) Let $x(t) = x_0(t) + u(t)$ be a new solution for the same system under slightly different initial conditions. Assuming that $u \ll x_0$ show that u satisfies the following equation

$$\ddot{u} + (p^2 + 3\mu x_0^2)u = 0$$ [20%]

(c) By substituting the value of $x_0(t)$ in (b), show that the equation can be transformed to the Mathieu equation, and obtain the value of the excitation frequency ω for which the system can be excited parametrically. [50%]

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4C7 2019 Numerical answers

1. (a) $S_{vv}(\omega) = (A/\pi)\alpha V/(\omega^2 + \alpha^2 V^2)$

   (b) $S_{rr}(\omega) = S_{vv}(\omega) M^2 \omega^4 / [(K - M\omega^2)^2 + (C\omega)^2]$

   (c) $\sigma_F^2 = (A/C)K^3 \alpha V/(\omega_n^2 + \alpha^2 V^2)$

2. (a) $P_f = 0.0315$

   (b) $P_f = 1.19 \times 10^{-7}$

   (c) $\mathrm{E}[D] = 0.329$

3. (b) $x = A\cos t - (\varepsilon A^5/16)[3\sin 3t + (1/3)\sin 5t]$, $A^4 = 3.$

4. (a) $(-\omega^2 + p^2 + 3\mu A^2/4)A = K$.

   (c) $\ddot{u} + (P^2 + \varepsilon\cos\Omega t)u = 0$, $P^2 = p^2 + 3\mu A^2/2$, $\varepsilon = 3\mu A^2/2$, $\Omega = 2\omega$

   $\omega = \sqrt{p^2 + 3\mu A^2/2}$