Version DC/4

EGT3
ENGINEERING TRIPOS PART IIB

Friday 24 April 2015 14:00 to 15:30

Module 4C8

APPLICATIONS OF DYNAMICS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C8 datasheet, 2014 (5 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A ‘coned’ railway wheelset with effective conicity ε, average wheel radius r and track gauge $2d$ is moving along a straight track at steady speed u. It has small lateral tracking error y and small yaw angle θ. The coefficients of both lateral and longitudinal creep of the wheels are C.

(a) Show that the net lateral force Y and net moment N acting on the wheelset due to the creep forces are given by:

$$Y = 2C\left(\theta + \frac{\dot{y}}{u}\right)$$

$$N = 2dC\left(\frac{\varepsilon\, y}{r} - \frac{d\dot{\theta}}{u}\right)$$

and indicate their directions on a sketch of the wheelset. State your assumptions. [50%]

(b) Figure 1 shows a special railway vehicle with a central wheelset that measures the roughness profile of the track. The central wheelset, which may be assumed massless, is attached to the vehicle body through a suspension system which has lateral stiffness k (restoring force per unit displacement) and yaw stiffness ka^2, (restoring moment per unit angle), where a is a constant. The front and rear wheelsets may be assumed to track perfectly, with zero lateral tracking error and zero yaw angle.

(i) Derive an equation for the motion of the central wheelset. [30%]

(ii) Find an expression for the wavelength of the hunting motion. Compare it with the hunting wavelength of a free wheelset. [10%]

(iii) Find an expression for the damping ratio of the hunting motion and sketch a graph of its variation with k. [10%]

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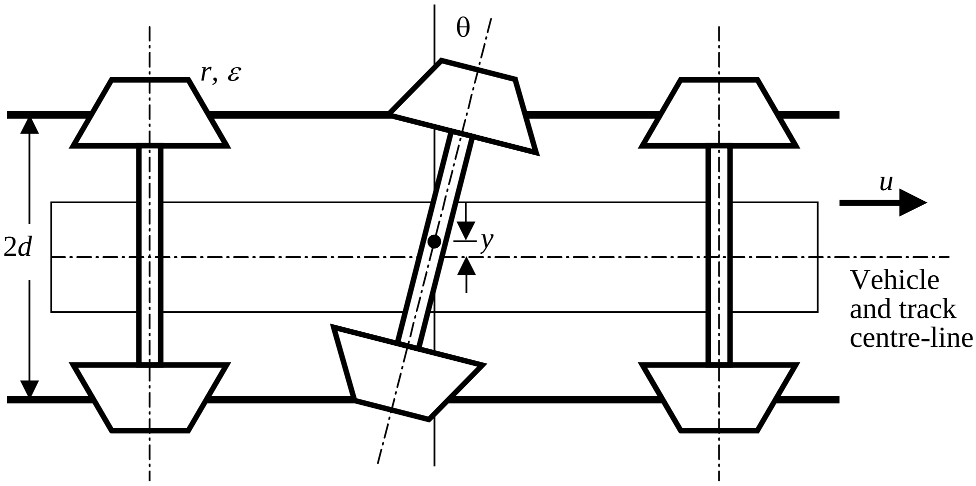

Fig. 1

2 A wheel rolls at yaw angle δ to the direction of motion of its axle. It is assumed that the contact conditions can be modelled using the 'brush' model with lateral stiffness per unit area K_y and a rectangular contact area of length $2L$ and width $2h$. The coefficient of friction is μ, the total normal force is Z and the contact pressure may be assumed constant over the contact area.

(a) Determine the realigning moment N, assuming that δ is small and that no micro-slip occurs. Explain what is meant by the 'pneumatic trail' and determine its value for small δ. [30%]

(b) For larger δ when micro-slip occurs, the realigning moment can be written in non-dimensional form as:

$$\frac{N}{\mu Z L} = \frac{1}{4\lambda}\left(\frac{1}{3\lambda} - 1\right), \quad \text{where} \quad \lambda = \frac{4L^2 h K_y}{\mu Z}\delta.$$

Explain how this formula is derived and show that it is consistent with the value of N for small δ from Part (a). [20%]

(c) Sketch the variation of N with δ, showing important features. Find the maximum value of N and the corresponding value of δ. Explain why this maximum occurs. [30%]

(d) Discuss briefly the relative importance of the realigning moment in simple models of vehicle dynamics. [20%]

3 The effect of the sun's gravitational field on the orbit of an artificial Earth satellite is to be investigated. The orbit is described by the usual polar co-ordinates (r, θ) with the origin at the centre of the Earth. The sun may be regarded as a fixed point of mass M a distance R from this origin in the direction $\theta = 0$.

(a) Assuming that $R >> r$ everywhere in the orbit, show that the local gravitational potential is given approximately by

$$U = \frac{mG}{r} + \frac{MG}{R}\left(1 + \frac{r}{R}\cos\theta\right)$$

where m is the mass of the Earth. Assume that the earth is a fixed point mass.

Hence obtain the approximate equation for radial motion for the satellite, and show that its equation of transverse motion can be written as

$$\frac{d}{dt}\left(r^2\dot{\theta}\right) \approx -\frac{MGr}{R^2}\sin\theta$$ [40%]

(b) When the effect of the sun is neglected, the satellite's orbit is circular with radius a and (constant) angular velocity ω. Now assume that the effect of the sun's potential can be modelled by letting $r = a + \delta(t)$ and $\theta = \omega t + \phi(t)$ with $|\delta| << a$ and $|\phi| << 1$ and hence solve the transverse equation of motion to show that

$$r^2\dot{\theta} \approx h_0 + \frac{MGa}{R^2\omega}\cos\omega t$$

where $h_0 = a^2\omega$. [30%]

(c) When the above result is combined with the corresponding result for radial motion, the following differential equation in δ is obtained:

$$\ddot{\delta} + \omega^2\delta = \frac{3MG}{R^2}\cos\omega t$$

Obtain the general solution for this equation, and comment briefly on its significance for the orbits of Earth satellites. [30%]

(TURN OVER

4 A deep space probe is to be launched from a space vehicle which is orbiting the Earth in a circular orbit of radius r_o. The probe is 'fired' explosively through a tube on the launch vehicle which is pointing in the direction of its travel, such that the velocity of the probe increases whilst that of the launch vehicle decreases.

(a) If the probe has an absolute velocity v when it leaves the tube, find expressions for the major axis and eccentricity of its subsequent orbit in terms of r_0, v and μ (where μ has its usual meaning, as defined on the data sheet). [50%]

(b) It is intended that the new velocity of the launch vehicle should be such that it is able to glide smoothly back to Earth, i.e. that the perigee of its subsequent orbit should lie on the Earth's surface. If the probe is to be given a starting velocity such that it can just escape the Earth's gravitational field, and if the mass of the launch vehicle is three times that of the probe, what height should be chosen for the initial circular orbit? [50%]

Treat the Earth as a uniform sphere, and neglect any effects of its atmosphere.

END OF PAPER

ANSWERS

1 (b) (i) $\ddot{y} + \left[\dfrac{uk\left(d^2 + a^2\right)}{2d^2C}\right]\dot{y} + \left[\dfrac{u^2}{d}\left(\dfrac{\varepsilon}{r} + \dfrac{a^2k^2}{4dC^2}\right)\right]y = 0$; (ii) $\lambda = \dfrac{2\pi}{\sqrt{\dfrac{\varepsilon}{dr} + \dfrac{a^2k^2}{4d^2C^2}}}$

(iii) $\zeta = \dfrac{\left(1 + a^2/d^2\right)k}{4C\sqrt{\dfrac{\varepsilon}{dr} + \dfrac{a^2k^2}{4d^2C^2}}}$

2 (a) $N = -\dfrac{4}{3}L^3hK_y\delta$; $L/3$;

(b) At $\lambda_{\lim} = \dfrac{1}{2}$, both solutions give $N = -\dfrac{1}{6}\mu ZL$ and $\dfrac{dN}{d\delta} = -\dfrac{4}{3}L^3hK_y$

(c) $N_{\max} = -\dfrac{3}{16}\mu ZL$ at $\lambda = \dfrac{2}{3}$

3 (c) $\delta = A\cos\omega t + B\sin\omega t + \dfrac{3MG}{2\omega R^2}t\sin\omega t$

4 (a) $a = \dfrac{\mu r_0}{2\mu - r_0v^2}$, $e = \dfrac{v^2r_0}{\mu}$; (b) $0.6921r_E$ = 4414 km.