EGT3
ENGINEERING TRIPOS PART IIB

Thursday 28 April 2016 9.30 to 11

Module 4C8

APPLICATIONS OF DYNAMICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: Module 4C8 data sheet (5 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Figure 1 shows a side view of an idealized model of a tandem-axle suspension of a heavy truck. The two closely-spaced axles are connected by a light rigid beam of length $2a$. The suspension, with stiffness k and damping c, is connected by a frictionless pivot to the centre of the beam. The centre of the beam has vertical displacement z_u. The upper end of the suspension is connected to the sprung mass m_s which has vertical displacement z_s. Inputs are the road displacements z_1 and z_2 at the leading and trailing axles. The wheels and tyres are assumed to be rigid and to follow the road displacement without loss of contact. Parameter values are: m_s = 10,000 kg; k = 617 kN m^{-1}; c = 47 kN s m^{-1}; a = 1 m; and U = 10 m s^{-1}.

The vehicle travels with speed U in the direction shown over a sinusoidal road profile of wavelength λ and unit amplitude displacement. The displacement input z_2 is a delayed version of the displacement input z_1. The amplitude of z_u is given by:

$$|z_u| = \left|\cos\left(\frac{2\pi a}{\lambda}\right)\right| \tag{1}$$

(a) Sketch and annotate a graph of the amplitude of z_u as a function of the spatial frequency $2a/\lambda$ for the range $0 < 2a/\lambda < 2$. With reference to this graph, explain the meaning of the term 'wheelbase filtering' in relation to vehicle vibration. Your explanation should mention the significance of the vehicle's speed and the distance between the axles. [25%]

(b) Sketch and annotate a graph of the magnitude of the transfer function from z_1 to z_s over the frequency range 0 Hz to 10 Hz, indicating clearly the frequencies at which the magnitude is zero. [35%]

(c) Derive expression (1). [40%]

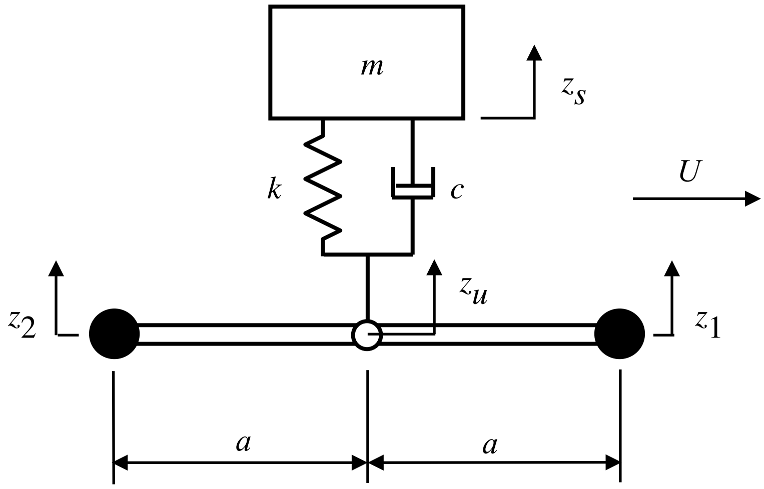

Fig. 1

2 (a) The vertical profiles of the left and right hand wheel tracks, z_L and z_R, distance $2T$ apart on a uniformly rough road surface can be defined in terms of an average vertical displacement z_V and a roll displacement z_ϕ as shown in Fig. 2.

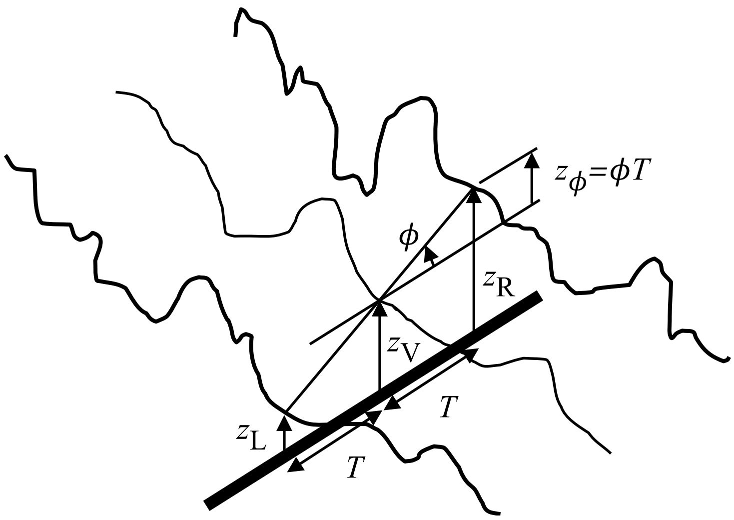

Fig. 2

The mean square spectral densities of z_V and z_ϕ can be related using $S_{z_\phi}(n) = |G(n)|^2 S_{z_V}(n)$, where $|G(n)|^2 = n^2 \left(n_c^2 + n^2\right)^{-1}$ and n_c is a 'cut-off' wavenumber. Derive expressions for $S_{z_V}(n) / S_{z_{L,R}}(n)$ and $S_{z_\phi}(n) / S_{z_{L,R}}(n)$, where $S_{z_{L,R}}(n)$ is the mean square spectral density of z_L or z_R, and sketch these as a function of wavenumber n. [40%]

(b) In the study of vehicle vibration in the roll-plane, lateral tyre behaviour can be represented by a spring k_{lat} and a damper c_{lat} connected in series to ground. By considering tyre slip angle, show that

$$k_{lat} = \frac{C}{L_{relax}} \quad \text{and} \quad c_{lat} = \frac{C}{U}$$

where C is the cornering stiffness, L_{relax} is the relaxation length of the tyres and U is the forward speed of the vehicle. [40%]

(c) With reference to your answers to part (a) and part (b), and considering vibration only in the roll-plane, explain how the mean square spectral densities of bounce acceleration and roll acceleration, measured at the centre of the sprung mass of a vehicle, are likely to be affected by the speed of the vehicle. [20%]

3 (a) Show how the standard equations for radial and tangential acceleration

$$a_r = \ddot{r} - r\dot{\theta}^2 \quad \text{and} \quad a_t = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

can be used to express Newton's equations of orbital motion

$$r^2\dot{\theta} = h \quad \text{and} \quad r\dot{\theta}^2 - \ddot{r} = \frac{\mu}{r^2}$$

where h and μ are positive constants, whose physical significance should be explained. [20%]

(b) Show how Newton's equations of orbital motion can then be used to derive the differential equation

$$\frac{d^2u}{d\theta^2} + u = \frac{\mu}{h^2}$$

where $u = 1/r$. [20%]

(c) Write down a general solution for the differential equation above, and show how this can be reduced to the standard equation for a Keplerian orbit, namely

$$r = \frac{a\left(1 - e^2\right)}{1 + e\cos\theta}$$

where a and e are constants, whose physical meanings should be explained. [20%]

(d) Use your results to prove Kepler's third law, namely that the period of a Keplerian orbit is proportional to $a^{3/2}$, but is independent of h and e. [20%]

(e) Observations show that planets in the solar system do not exactly obey Kepler's laws as they orbit around the Sun. List the main reasons for this and discuss how Kepler was able to postulate his laws despite these discrepancies. [20%]

(TURN OVER

4 (a) Explain the physical reason for the J_2 term in the expression for the external potential of the Earth, given on the data sheet. [10%]

(b) A thin ring of matter, with radius R and total mass m, is centred at the origin of a system of spherical polar co-ordinates with its axis of symmetry coinciding with the polar axis, as shown in Fig. 3. Show that the gravitational potential at point P, whose co-ordinates are (r, θ, 0) can be written as:

$$U = \frac{mG}{2\pi} \int_0^{2\pi} \frac{d\phi}{\sqrt{r^2 + R^2 - 2rR\sin\theta\cos\phi}}$$ [30%]

(c) For the case where $r >> R$, use the binomial theorem to evaluate this integral approximately, keeping terms up to the order R^2/r^2. [30%]

(d) The Earth can be modelled as a uniform sphere of mass M and radius R plus an additional ring of the type investigated in part (b) around its equator, such that $M_{\text{Earth}} = M + m$. Use the expression derived in part (c) and the value of J_2 given in the data sheet to determine what fraction of the Earth's total mass should be modelled into the ring, to simulate the external potential of the Earth. [30%]

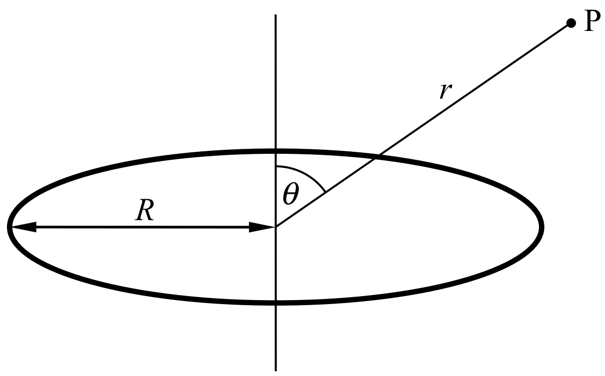

Fig. 3

END OF PAPER

Answers

2 (a) $\frac{n_c^2+n^2}{n_c^2+2n^2}$, $\frac{n^2}{n_c^2+2n^2}$

4 (c) $U = \frac{mG}{r}\left(1 - \frac{R^2}{2r^2} + \frac{3}{4}\frac{R^2}{r^2}(\sin\theta)^2\right)$

4 (d) 2.16×10^{-3}