Version DC/2

EGT3
ENGINEERING TRIPOS PART IIB

Wednesday 24 April 2019   2 to 3.40

Module 4C8

VEHICLE DYNAMICS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C8 datasheet, 2017 (3 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Figure 1 shows a railway wheelset with effective conicity ε, average wheel radius r and track gauge $2d$, moving along a horizontal track with radius of curvature R at steady speed u. It has small lateral tracking error y and small yaw angle θ. The coefficients of *both* lateral and longitudinal creep of the wheels are C.

(a) Show that the net lateral force Y and net yaw moment N acting on the wheelset due to the creep forces are given by:

$$Y = 2C\left(\theta + \frac{\dot{y}}{u}\right)$$

$$N = 2dC\left(\frac{\varepsilon\, y}{r} - \frac{d\dot{\theta}}{u} - \frac{d}{R}\right)$$

and indicate their directions on a sketch of the wheelset. State your assumptions. [30%]

(b) Figure 2 shows a bogie, comprising two such wheelsets, connected by a rigid frame at a spacing of $2a$. The bogie runs along a track that is near to straight, with varying radius of curvature R.

(i) Show that the lateral tracking error y of the centre of the bogie can be written

$$\ddot{y} + \left(\frac{\varepsilon}{dr}\frac{u^2}{\left(1+\dfrac{a^2}{d^2}\right)}\right)y = \left(\frac{u^2}{1+\dfrac{a^2}{d^2}}\right)\frac{1}{R}$$ [40%]

(ii) Find an expression for the wavelength of the hunting motion on straight track. Compare it with the hunting wavelength of a free wheelset. [10%]

(iii) The track has a lateral displacement that varies sinusoidally with distance with wavelength L and amplitude Δ. Write an expression for the curvature of the track and hence determine, and sketch a graph of, the amplitude of the lateral tracking error of the bogie as a function of the reciprocal of the wavelength L, showing salient values. [20%]

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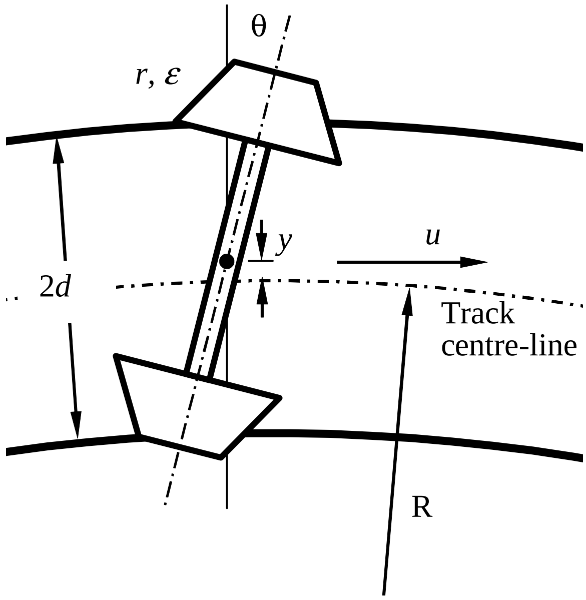

Fig. 1

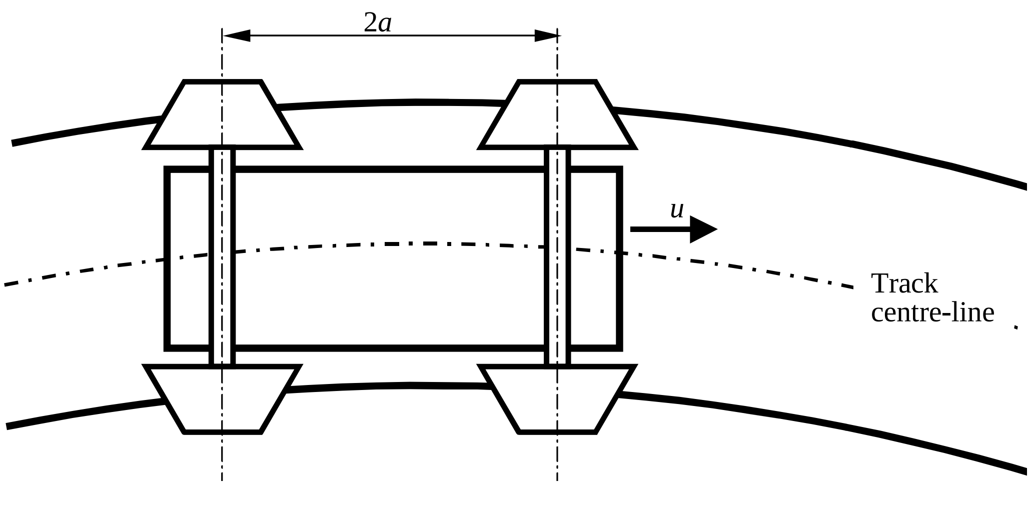

Fig. 2

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2 Figure 3 shows the single-sided mean square spectral density (MSSD) of the vertical displacement along one wheel-track of a randomly rough road surface. It also shows three analytical single-sided MSSDs of the form

$$S_{z_r}(n) = \kappa n^{-w}.$$

(a) Explain the significance of the parameters κ, n and w, and explain the significance of the term 'single-sided'. [10%]

(b) Estimate the values of κ and w for the measured MSSD shown in Fig. 3. [10%]

(c) The frequency response function of a two-degree-of-freedom quarter-car model, relating vertical road displacement input to sprung mass vertical acceleration output, is denoted as

$$H(j\omega) = \frac{\ddot{z}_s(j\omega)}{z_r(j\omega)}.$$

It is proposed to calculate analytically the mean square acceleration of the sprung mass $\mathrm{E}\left[\ddot{z}_s^2\right]$ in response to the vehicle travelling at speed U along the randomly rough road surface. The analytical solution requires the mean square to be expressed in the form:

$$\mathrm{E}\left[\ddot{z}_s^2\right] = S_0 \int_{-\infty}^{\infty} \left|G(j\omega)\right|^2 \mathrm{d}\omega$$

where S_0 is the double-sided MSSD of a white noise signal and $G(j\omega)$ is a frequency response function expressed as a quotient of two polynomials in $j\omega$.

(i) Derive an expression for the analytical MSSD of the road displacement z_r as a function of angular temporal frequency ω (in rad s^{-1}) instead of wavenumber n (in cycle m^{-1}). You may make use of the following relationship:

$$\mathrm{E}\left[z_r^2\right] = \int_0^{\infty} S_{z_r}(n)\,\mathrm{d}n = \int_0^{\infty} S_{z_r}(\omega)\,\mathrm{d}\omega$$ [35%]

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(ii) Derive an expression for S_0 in terms of κ and U, stating any approximations involved. [30%]

(iii) Derive an expression for $G(\mathrm{j}\omega)$ in terms of $H(\mathrm{j}\omega)$. [15%]

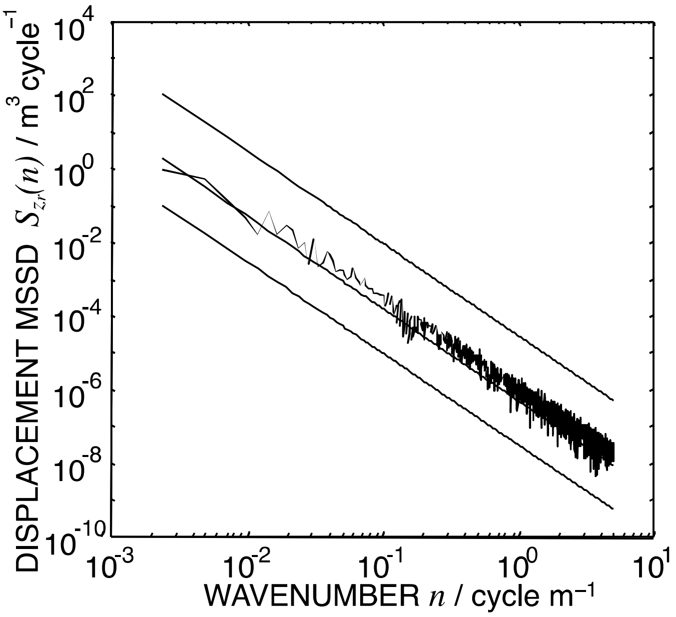

Fig. 3

3 Figure 4 shows an idealised 'pitch-plane' model of a two-axle truck with a flexibly mounted cab above the front axle. The model consists of two parts: a massless rigid beam AB of length *l*, and a rigid beam AP of length *h* with point mass *m* at P. The two beams are pivoted at A and a torsional spring with stiffness *k* and a torsional damper with damping *c* resist relative rotation of the two beams. The vertical displacement inputs to the front and rear axles at A and B are $z_1(t)$ and $z_2(t)$. The horizontal displacement of P relative to A is *y*, which relates to discomfort experienced by occupants of the cab.

(a) Derive an expression for the frequency response function relating the two displacement inputs $z_1(j\omega)$ and $z_2(j\omega)$ to the displacement output $y(j\omega)$. Assume that any vertical forces acting on mass *m* do not couple into longitudinal or pitch motions. [30%]

(b) Show that the frequency response function found in part (a) corresponds to 'Case (c)' in the Mechanics Data Book and find expressions for the natural frequency and the excitation. [15%]

(c) If the vehicle travels along a single wheel track profile at speed *U* in the positive *y* direction, derive an expression for the frequency response function relating $z_1(j\omega)$ to $z_2(j\omega) - z_1(j\omega)$. Hence find an expression for the frequencies at which $z_2(j\omega) - z_1(j\omega)$ is zero. [30%]

(d) Using results from parts (a), (b) and (c) sketch the magnitude of the frequency response function from $z_1(j\omega)$ to $y(j\omega)$. Annotate the sketch with expressions for the salient points and suggest a strategy for selecting stiffness *k* to minimise discomfort. [25%]

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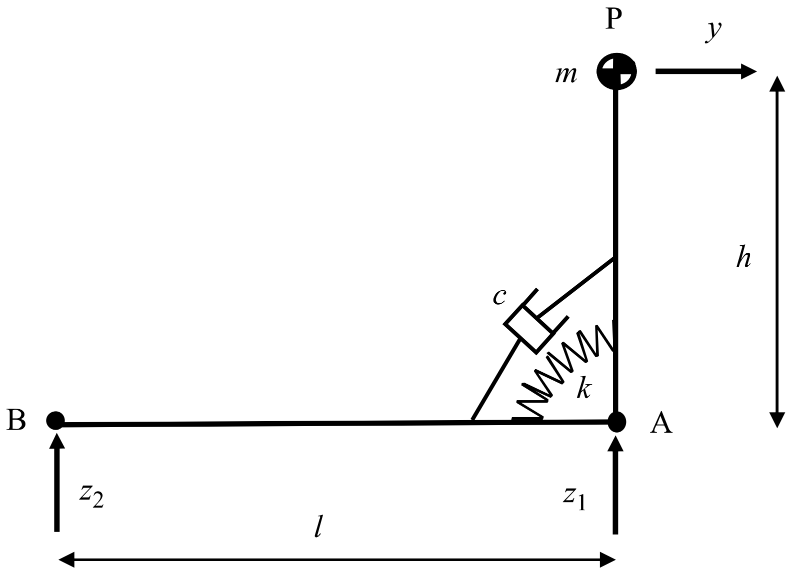

Fig. 4

4 A 'bicycle' model of a vehicle, with freedom to sideslip with velocity v and yaw at rate Ω, is shown in Fig. 5. The vehicle moves at steady forward speed u. It has mass m, yaw moment of inertia I, and lateral creep coefficients C at both the front and rear tyres. The lengths a and b and the angle δ are defined in the figure.

(a) Show that the equations of motion in a coordinate frame rotating with the vehicle are given by:

$$m\left(\dot{v}+u\Omega\right)+2C\frac{v}{u}+\left(a-b\right)C\frac{\Omega}{u}=C\delta$$
$$I\dot{\Omega}+\left(a-b\right)C\frac{v}{u}+\left(a^2+b^2\right)C\frac{\Omega}{u}=aC\delta$$

and state your assumptions. [30%]

(b) An autonomous vehicle is found to have a critical speed above which its motion becomes unstable. It is proposed to eliminate this problem by steering the front wheels in response to yaw rate, so that $\delta=-K\Omega$, with K a positive constant.

(i) Derive a characteristic equation that can be used to analyze the stability of the vehicle. [30%]

(ii) Determine the conditions for which forward motion of the vehicle is stable. Sketch a graph of the variation of the critical speed with K, showing salient values. Compare the critical speed of the controlled vehicle to that of the uncontrolled vehicle. [40%]

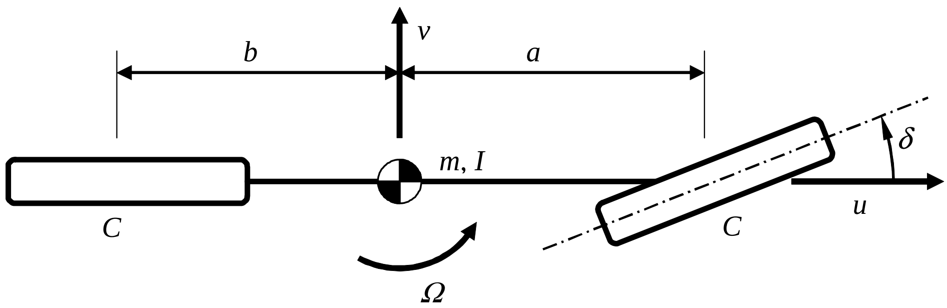

Fig. 5

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