Version NAF/2

EGT3
ENGINEERING TRIPOS   PART IIB

Wednesday 22 April 2015   9.30 to 11

Module 4C9

CONTINUUM MECHANICS

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C9 datasheet (2 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) The governing relations in elastostatics are of the form:

$$\sigma_{ij,j} = 0 \quad , \qquad \sigma_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + 2\mu \varepsilon_{ij} \quad , \qquad \text{and } \varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$$

where the various symbols take on their usual meanings. State the underlying assumptions for these three relations. What additional condition must be met to ensure that the solution is unique? [30%]

(b) A visco-elastic solid is represented by a Maxwell element, comprising a linear spring of modulus E in series with a linear dashpot of viscosity η. The solid is subjected to an axial strain

$$\varepsilon(t) = 0 \qquad \text{for } t < 0$$
$$\varepsilon(t) = A\sin(\omega t) \qquad \text{for } t \geq 0$$

Calculate the stress $\sigma(t)$, and sketch it in relation to $\varepsilon(t)$. [30%]

(c) The Drucker-Prager yield criterion states that incompressible plastic flow occurs when the combination of von Mises effective stress σ_e and hydrostatic stress $\sigma_h = \frac{1}{3}\sigma_{kk}$ attains a yield strength σ_Y such that

$$\sigma_Y = \sigma_e + C\sigma_h$$

where C is a constant. Explain whether Drucker's postulates are satisfied or not. [40%]

2 (a) Evaluate the following using indicial notation:

(i) $\nabla \times \nabla \phi$ where ϕ is a scalar function of position

(ii) $\left(\tilde{a} \times \tilde{b}\right) \cdot \left(\tilde{c} \times \tilde{d}\right)$ where $\tilde{a}, \tilde{b}, \tilde{c}$ and $\tilde{d}$ are vectors.

(iii) $\partial\beta/\partial A_{kl}$ for $\beta = A_{ij}A_{ij}$ [30%]

(b) A rectangular punch of width *w* indents a rigid, ideally plastic solid of shear yield strength *k*, as shown in Fig. 1. The indenter moves at a velocity *v* at an inclination of $\pi / 4$ to the surface of the solid, and plane strain conditions are assumed. A possible collapse mechanism is given by an array of 5 identical sliding blocks, with each block in the form of an isosceles triangle, as shown.

(i) Assuming frictionless contact, estimate the indentation load per unit thickness into the page. [20%]

(ii) Now assume that the indenter adheres to the surface of the solid. Obtain a revised value for the indentation load. [20%]

(iii) An alternative collapse response for the sticking case involves relative sliding of the punch on the half-space such that the tangential velocity jump at the punch interface equals $v / \sqrt{2}$. Obtain a new estimate for the indentation load. Explain whether this mode is more likely than that given above in (b) part (ii). [30%]

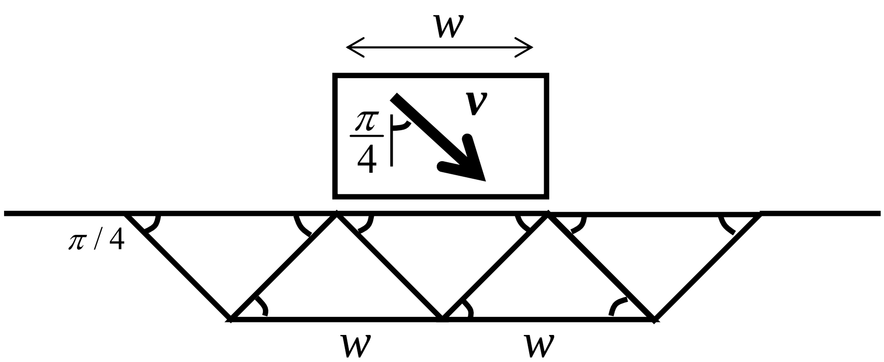

Fig. 1

3 (a) What are the differences between the slip line field method and the upper bound method? When do they coincide? [30%]

(b) A light cantilever beam of depth $2h$ and length L is shown in Fig. 2. It is loaded by a uniform distribution of normal traction q_1 and q_2 on the top and bottom surfaces, respectively. A candidate Airy's stress function for representing the stress field in the cantilever is

$$\Phi = \frac{A}{6}x^2y^3 + \frac{B}{3}x^3y + \frac{C}{2}x^2y + \frac{D}{6}y^3 + \frac{E}{20}y^5$$

(i) Show that this choice of stress function requires that $q_1 = -q_2$. Obtain expressions for the coefficients A, B, C and E in terms of q_2. [30%]

(ii) Express the components of stress σ_{xx}, σ_{xy}, σ_{yy} in terms of q_2. [15%]

(iii) Obtain the value of D such that there is no net moment on the beam at $x = 0$. What is the magnitude of the shear force at this location? [25%]

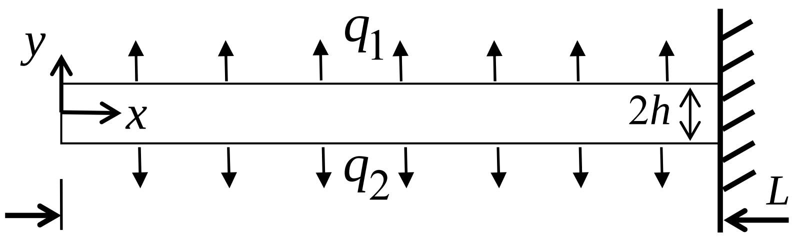

Fig. 2

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