EGT3
ENGINEERING TRIPOS PART IIB

Monday 18 April 2016 14:00 to 15:30

Module 4D6

DYNAMICS IN CIVIL ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number __**not**__ your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4D6 Dynamics in Civil Engineering data sheets (4 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Figure 1(a) shows a model of a rail from a railway track, based on an Euler beam of bending stiffness EI and mass per unit length m. The beam is assumed to be simply supported at the locations of the railway sleepers, which have a uniform spacing L.

(a) Derive an estimate for the fundamental 'pinned-pinned' natural frequency of the rail, assuming a mode shape similar to that illustrated in Fig. 1(b). Evaluate this for the case of $EI = 6.42$ MN m^2, $m = 60$ kg m^{-1} and $L = 0.6$ m. [40%]

(b) A train with a defective wheel applies the idealised dynamic load shown in Fig. 2, which impacts the rail with each revolution of the wheel. By considering a single impact at the worst possible location, estimate the peak displacement of the rail due to the excitation of this pinned-pinned mode. Comment on any short comings of the simplifications in this model. [40%]

(c) In practice, the rail is supported on rubber pads between the base of the rail and each sleeper. Explain how this might change:

(i) the pinned-pinned frequency calculated in part (a);

(ii) the displacement estimate calculated in part (b). [20%]

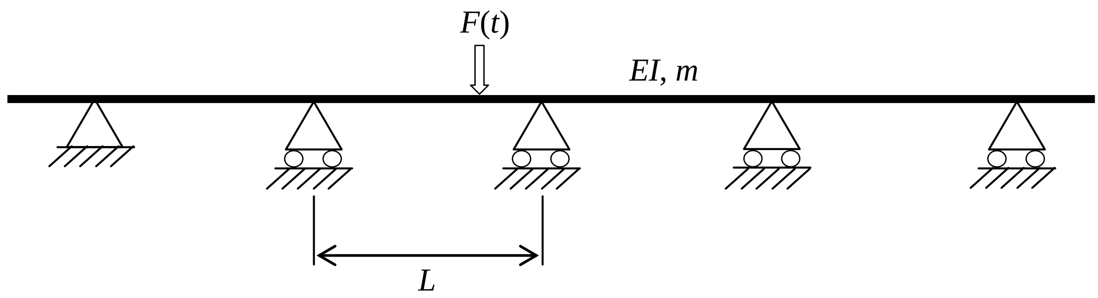

Fig. 1(a)

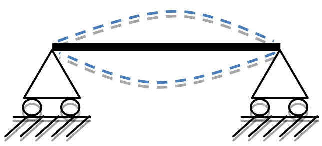

Fig. 1(b)

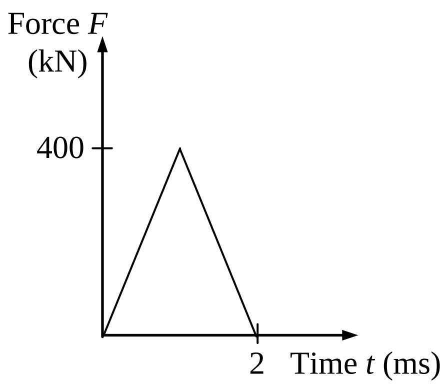

Fig. 2

2 (a) Write the assumptions made to create a tripartite response spectrum, and state when these assumptions are reasonable. [15%]

(b) Explain why ductility is desired in seismic design. [15%]

(c) A two-storey structure is shown in Fig. 3. Assume that the floors are rigid, and that all columns and braces are steel and are pinned at their ends. The top storey braces have a cross-sectional area of 100 mm^2, while the bottom storey braces have a cross-sectional area of 200 mm^2.

(i) Find the natural frequencies of the system. [20%]

(ii) Assume the design PGA is 0.2g, and the elastic design spectrum is shown in Fig. 4. Neglecting any contribution from the second mode, calculate the maximum inter-storey drift and the required ductility. [50%]

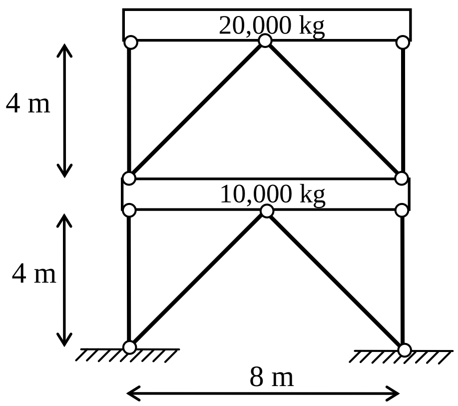

Fig. 3

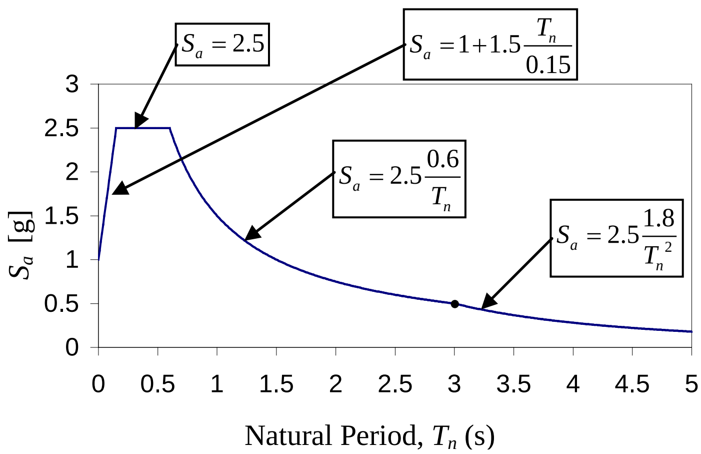

Fig. 4

3 A reinforced concrete water tank is to be constructed at a site in Abu Dhabi as shown in Fig. 5. The column forming the tower is 10 m long and has a flexural stiffness of 30 MN m^2. The water tank is a square in the plan view with the outside dimensions of 3 m and a wall thickness of 0.25 m. The density of concrete is 2400 kg m^{-3}. The site has a deep sand layer that has unit weight of 14.3 kN m^{-3} and a void ratio of 0.75. The raft foundation is a 2 m × 2 m square in the plan view and is 0.5 m thick as shown in Fig. 5.

(a) Assuming that the water tank is empty and the foundation raft provides full fixity to the tower, calculate the natural frequency of the structure. You may assume that the mass of the water tank is lumped 11 m above the ground surface. [15%]

(b) Recalculate the natural frequency in Part a) if the water tank is full. You may assume that the water tank mass is lumped 11.5 m above the ground surface in this case. [15%]

(c) Consider a reference plane 1.5 m below the ground surface. The mass of the water tank, tower and raft is 50,950 kg when empty and 66,575 kg when full. Calculate the small-strain shear modulus of the soil and hence the rotational stiffness of the raft foundation for each of these cases. The Poisson's ratio for sand is 0.3 and its friction angle is 30^{o}. [25%]

(d) Suggest a suitable single degree of freedom discrete model for the rocking vibrations of the water tank-foundation soil system. Estimate the natural frequencies of this system for the cases when the water tank is empty and full. You may assume that the soil participating in rocking vibrations has a mass moment of inertia about the central axis A located on the reference plane of 18×10^{6} kg m^2, when the tank is empty, and 20×10^{6} kg m^2 when the tank is full. [25%]

(e) Comment on the importance of considering the soil-structure interaction in this problem. [20%]

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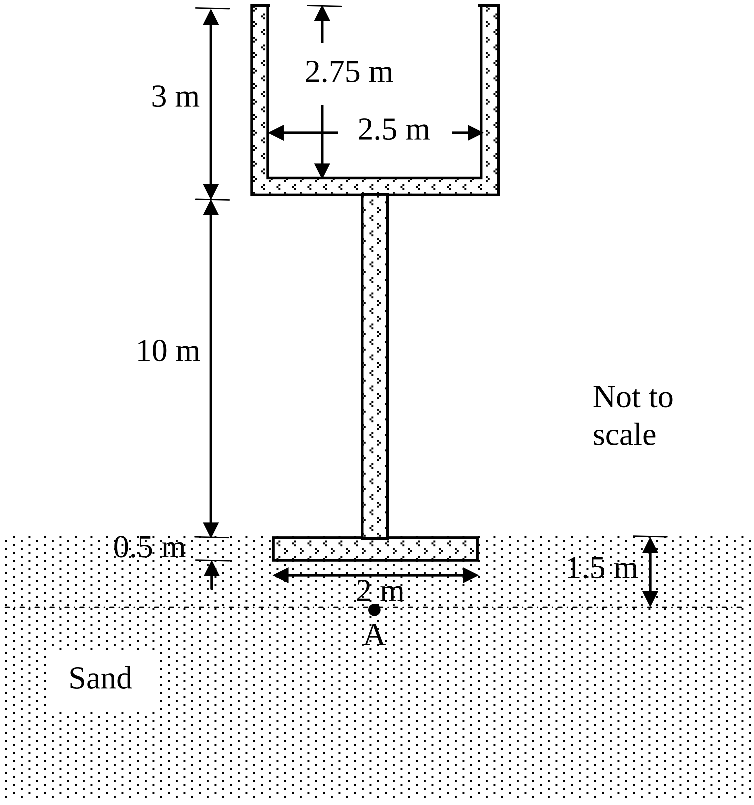


Fig. 5

(TURN OVER

4 A steel beam of circular hollow section has external diameter 193.7 mm and wall thickness 5.0 mm. It is used as part of a roof structure, acting as a simply-supported beam over a span of 8 m. The fraction of critical damping is estimated to be 0.5%. The following definitions may be of use:

The Strouhal Number $St = n_v D/U$, where n_v is the vortex shedding frequency (Hz), D is the cylinder diameter (m) and U is the flow velocity (m s^{-1}). For a circular cylinder, the critical Strouhal Number is 0.2.

The Scruton Number $Sc = 2\,\delta m/\rho D^2$, where the logarithmic decrement $\delta = 2\pi\xi$, with ξ the fraction of critical damping, m is the mass per unit length and ρ is the density of air (which may be taken as 1.25 kg m^{-3}). Dimensionless amplitudes $y_{\max}/D$ may be estimated as $1.5/Sc$.

(a) Determine the critical wind speed at which the beam would be expected to undergo vortex induced resonance in its fundamental mode. [30%]

(b) Estimate the amplitude of vibration at vortex-induced resonance. [10%]

(c) If the wall thickness were to be increased, what would be the likely effect on the fatigue life of the beam with respect to the vortex-induced vibrations? [20%]

(d) Describe two measures that may be taken to improve the fatigue life of the beam. [20%]

(e) Explain how the calculations would change if the beam were to be used underwater. [20%]

END OF PAPER

Module 4D6: Dynamics in Civil Engineering

Data Sheets

Approximate SDOF model for a beam

for an assumed vibration mode $\bar{u}(x)$, the equivalent parameters are

$$M_{eq} = \int_0^L m\,\bar{u}^2 dx \qquad K_{eq} = \int_0^L EI\left(\frac{d^2\bar{u}}{dx^2}\right)^2 dx \qquad F_{eq} = \int_0^L f\,\bar{u}dx + \sum_i F_i\,\bar{u}_i$$

Frequency of mode $u(x,t) = U\sin\omega t\,\bar{u}(x)$ $\quad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} \qquad \omega = 2\pi f$

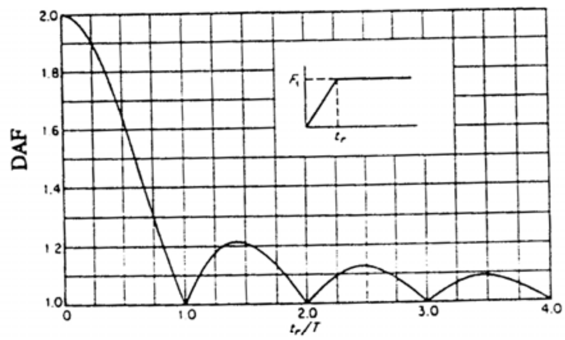
Modal analysis of simply-supported uniform beams

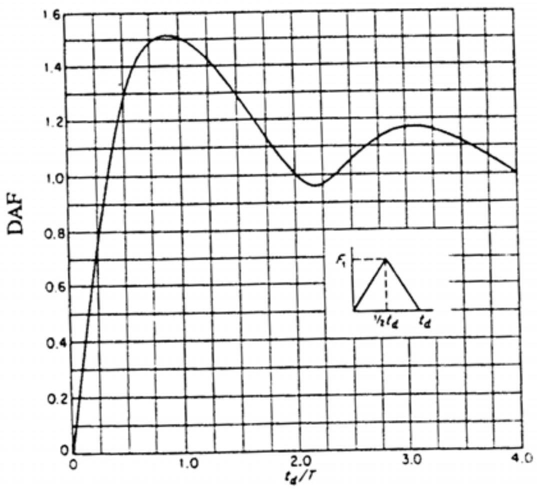
$$u_i(x) = \sin\frac{i\pi x}{L} \qquad M_{i\,eq} = \frac{mL}{2} \qquad K_{i\,eq} = \frac{(i\pi)^4 EI}{2L^3}$$

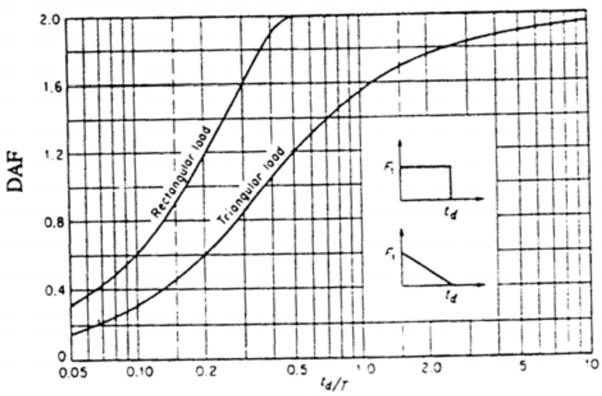
Ground motion participation factor

$$\Gamma = \frac{\int m\bar{u}dx}{\int m\bar{u}^2 dx}$$

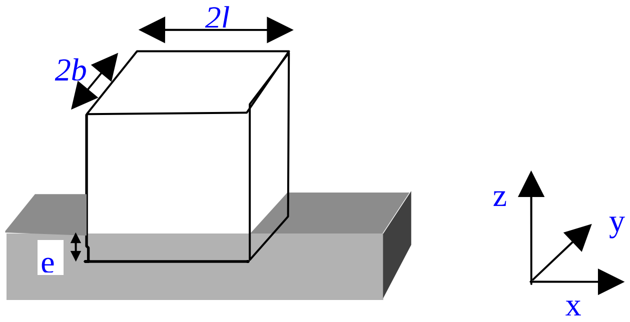
Dynamic amplification factors







Approximate relations for evaluating soil stiffness for an embedded prismatic structure of dimensions *2l* and *2b*, embedded to a depth *e* are:



$$K_{hx} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 2.4\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{hy} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 0.8\frac{l}{b} + 1.6\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{v} = \frac{Gb}{2-\nu}\left[3.1\left(\frac{l}{b}\right)^{0.75} + 1.6\right]\left[1 + \left(0.25 + \frac{0.25b}{l}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{rx} = \frac{Gb^3}{1-\nu}\left[3.2\frac{l}{b} + 0.8\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35+\frac{l}{b}}\left(\frac{e}{b}\right)^{2}\right)\right]$$

$$K_{ry} = \frac{Gb^3}{1-\nu}\left[3.73\left(\frac{l}{b}\right)^{2.4} + 0.27\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35+\left(\frac{l}{b}\right)^{4}}\left(\frac{e}{b}\right)^{2}\right)\right]$$

$$K_{tor} = Gb^3\left[4.25\left(\frac{l}{b}\right)^{2.45} + 4.06\right]\left[\left(1 + \left(1.3 + 1.32\frac{b}{l}\right)\left(\frac{e}{b}\right)^{0.9}\right)\right]$$

Unit weight of soil:

$$\gamma = \frac{(G_s + eS_r)\gamma_w}{1+e}$$

where e is the void ratio, S_r is the degree of saturation, G_s is the specific gravity of soil particles.

For dry soil this reduces to

$$\gamma_d = \frac{G_s \gamma_w}{1 + e}$$

Effective mean confining stress

$$p' = \sigma'_v \frac{(1 + 2K_o)}{3}$$

where σ'_v is the effective vertical stress, K_o is the coefficient of earth pressure at rest given in terms of Poisson's ratio ν as

$$K_o = \frac{\nu}{1 - \nu}$$

Effective stress Principle:

$$p' = p - u$$

Shear modulus of sandy soils can be calculated using the approximate relation:

$$G_{\max} = 100 \frac{(3 - e)^2}{(1 + e)} (p')^{0.5}$$

where p′ is the effective mean confining pressure in **MPa,** e is the void ratio and Gmax is the small strain shear modulus in **MPa**

Shear modulus correction for strain may be carried out using the following expressions;

$$\frac{G}{G_{\max}} = \frac{1}{1 + \gamma_h}$$

where

$$\gamma_h = \frac{\gamma}{\gamma_r}\left[1 + a \,.\, e^{-b\left(\frac{\gamma}{\gamma_r}\right)}\right]$$

'a' and 'b' are constants depending on soil type; for sandy soil deposits we can take

$$a = -0.2 \ln N$$

$$b = 0.16$$

where N is the number of cycles in the earthquake, γ is the shear strain mobilised during the earthquake and γ_r is reference shear strain given by

$$\gamma_r = \frac{\tau_{\max}}{G_{\max}}$$

where

$$\tau_{\max} = \left[\left(\frac{1 + K_o}{2} \sigma'_v \sin \phi' \right)^2 - \left(\frac{1 - K_o}{2} \sigma'_v \right)^2 \right]^{0.5}$$

Shear Modulus is also related to the shear wave velocity $\boldsymbol{v_s}$ as follows;

$$v_s = \sqrt{\frac{G}{\rho}}$$

where G is the shear modulus and ρ is the mass density of the soil.

Natural frequency of a horizontal soil layer f_n is;

$$f_n = \frac{v_s}{4H}$$

where v_s is shear wave velocity and H is the thickness of the soil layer.

SPGM
January, 2006

Q1

(a) 1427 Hz
(b) 0.32 mm

Q2

(c) (i) 1.7 Hz (ii) 5.5 Hz
(d) 0.030 m and 4.35

Q3

(a) 0.25 Hz
(b) 0.187 Hz
(c) 984.44 MNm/rad
(d) 1.0 Hz and 0.96 Hz

Q4

(a) 8.2 m/s
(b) 9.3 mm

JPT/MSPG/MJD/FAM