EGT3
ENGINEERING TRIPOS PART IIB

Friday 3 May 2019 09.30 to 11.10

Module 4D6

DYNAMICS IN CIVIL ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4D6 Dynamics in Civil Engineering Data Sheets (5 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A two storey building is planned to be constructed in Thessalonica, Greece. The building is modelled as a two-dimensional, rigid-jointed, linear elastic sway frame shown in Fig. 1. Its beams have a mass of 200 kg m^{-1}. The bending stiffness of the ground floor and first floor columns are EI = 6000 kN m^2 and EI = 4000 kN m^2 respectively.

(a) Which of the two mode shapes below, proposed as the fundamental mode for the building would you use and why? The displacement of the top floor has been normalised to unity.

$$\begin{bmatrix} 1 \\ 0.25 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 0.42 \end{bmatrix}$$

[30%]

(b) The building, initially at rest, is subject to a load pulse $F(t)$ applied to the 1st floor only. $F(t)$ is defined in Fig. 2. Calculate the maximum displacement of the top of the frame assuming vibration in the fundamental mode only. [30%]

(c) Obtain an improved estimate of the maximum dynamic displacement by considering the vibration response in both the first and second modes of the structure. [40%]

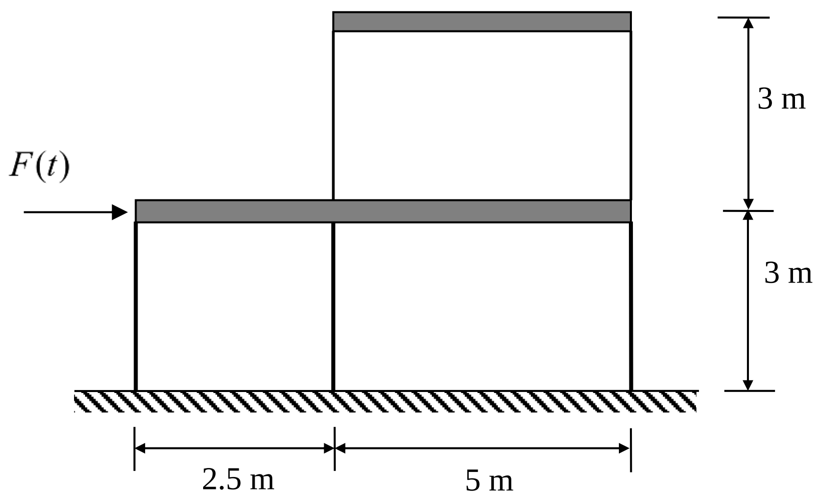

Fig. 1

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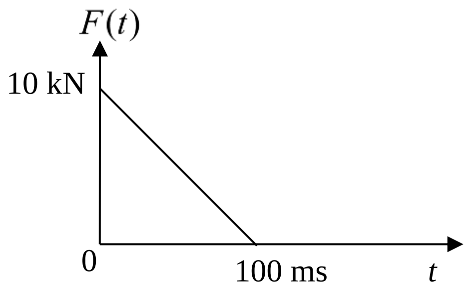

Fig. 2

2 (a) Explain what is meant by the terms *pseudo-velocity* and *pseudo-acceleration* as used in the construction of tripartite response spectra. [20%]

(b) Figure 3 shows a two-storey sway frame. Each storey has a mass of 3400 kg. Each column has flexural rigidity EI = 150 kN m^2. The column connections to the ground are pinned, and all other connections are fixed.

The in-plane sway modes have natural frequencies of 0.34 Hz and 1.46 Hz with associated mode shapes [1, 0.883] and [1, -1.13] respectively, these having been normalised to have unit displacement at the upper floor. The structure has 5% damping, and experiences an earthquake with the response spectrum shown in Fig. 4.

(i) Determine the maximum ground acceleration and displacement. [10%]

(ii) Determine the maximum column shear. [40%]

(c) The structure is being designed using the inelastic design spectra in Fig. 5 for an earthquake with peak ground acceleration of 0.4g. Considering only the first mode, determine the required ductility factor μ if the columns have a shear capacity of 2 kN. [30%]

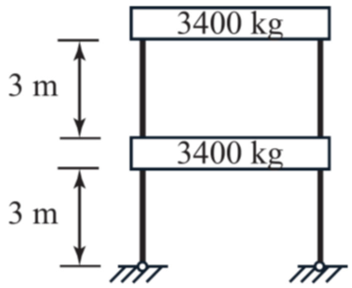

Fig. 3

Fraction of critical damping, ξ = 0, 2, 5, 10, 20%

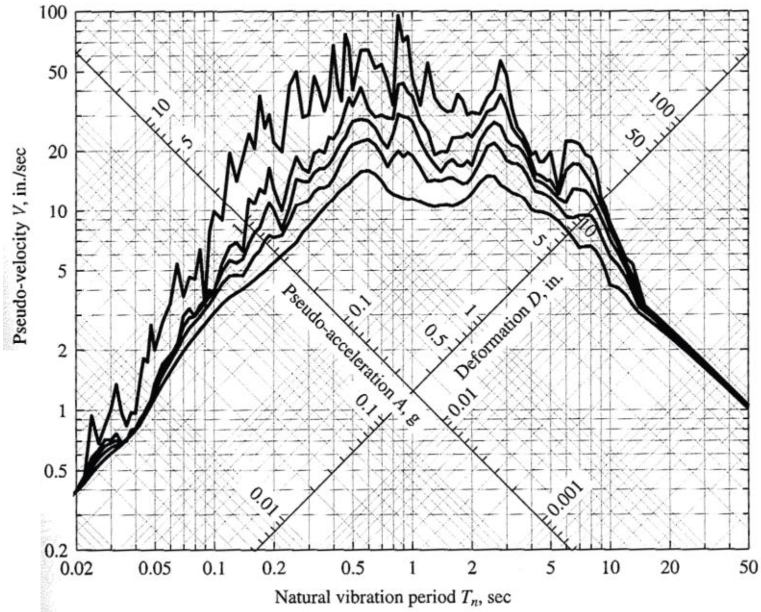

Fig. 4

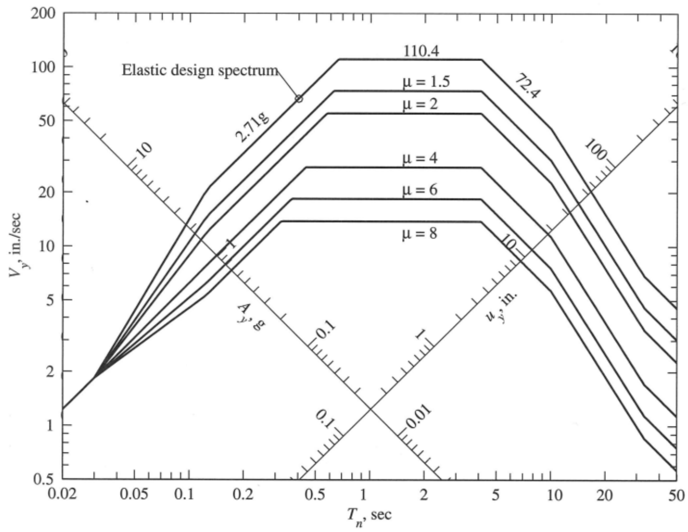

Fig. 5

3 (a) Explain why excess pore water pressures will be generated in loose, saturated sand when subjected to earthquake loading. [10%]

(b) Explain the terms '*full liquefaction'* and '*partial liquefaction'* and briefly state what dangers each of these can pose to structures. [10%]

(c) A crane structure is to be supported on a foundation block with dimensions of 6 m × 6 m × 4 m shown in Fig. 6, and is to be embedded to a depth of 4 m. This foundation is to be located in sandy soil that has a dry unit weight of 15 kN m^{-3} and a void ratio of 0.67. The expected direction of ground shaking is shown in Fig. 6. The shear wave velocity in this soil is measured as 150 m s^{-1} and the Poisson's ratio of the soil may be taken as 0.3. Assuming that the block foundation is rigid, determine the horizontal, vertical and rotational stiffness due to the soil. [30%]

(d) The mass of the foundation including the participating soil around it was determined to be 820,000 kg. The mass moment of inertia of the block foundation and participating soil around it was determined to be 4,000,000 kg m^2 about the expected axis of rotation. Determine the natural frequencies of vibration for the horizontal, vertical and rotational degrees of freedom for the block foundation. [20%]

(e) A strong earthquake was experienced by the block foundation. Due to the large cyclic strains and excess pore pressure generation, the shear modulus of the sand has reduced to 3 MPa. How does this affect the natural frequencies of vibration? Would you expect resonance of the block foundation under these conditions? What effects might this have on the crane structure? [30%]

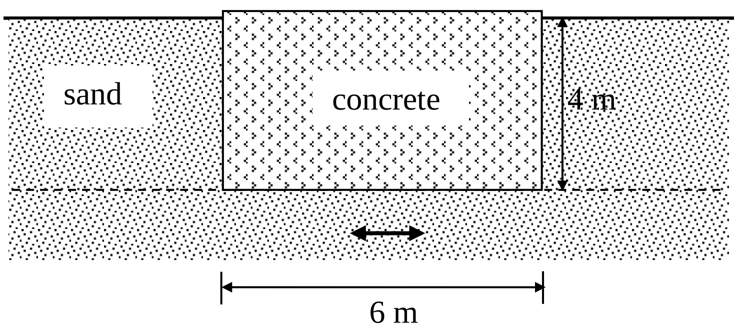

Fig. 6

4 (a) Explain why the response spectrum method that is used in earthquake engineering is not appropriate in wind engineering, and outline a method that is appropriate for determining the response of large flexible structures to excitation by wind buffeting. [50%]

(b) Describe the various theories of flutter, with particular reference to their use in long-span bridge design. [50%]

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Answers

4D6 Dynamics of Civil Engineering – Easter 2019

Q1 a) $\begin{bmatrix} 1 \\ 0.42 \end{bmatrix}$

b) 2.25 mm

c) 2.28 mm

Q2) b i) 8 in; $a \approx 0.32g$

b ii) 4.33 kN per column

c) $\mu \approx 4$

Q3) c) 1260 MN/m; 465 MN/m; 23493 MN-m/rad

d) 6.2 Hz; 3.8 Hz; 12.2 Hz

e) 1.8 Hz; 1.2 Hz; 3.6 Hz

Module 4D6: Dynamics in Civil Engineering

Data Sheets

Approximate SDOF model for a beam

for an assumed vibration mode $\bar{u}(x)$, the equivalent parameters are

$$M_{eq} = \int_0^L m\,\bar{u}^2 dx \qquad K_{eq} = \int_0^L EI\left(\frac{d^2\bar{u}}{dx^2}\right)^2 dx \qquad F_{eq} = \int_0^L f\,\bar{u}dx + \sum_i F_i\,\bar{u}_i$$

Frequency of mode $u(x,t) = U\sin\omega t\,\bar{u}(x)$ $\quad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} \qquad \omega = 2\pi f$

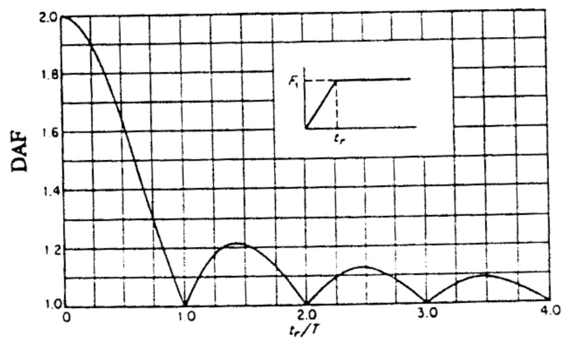
Modal analysis of simply-supported uniform beams

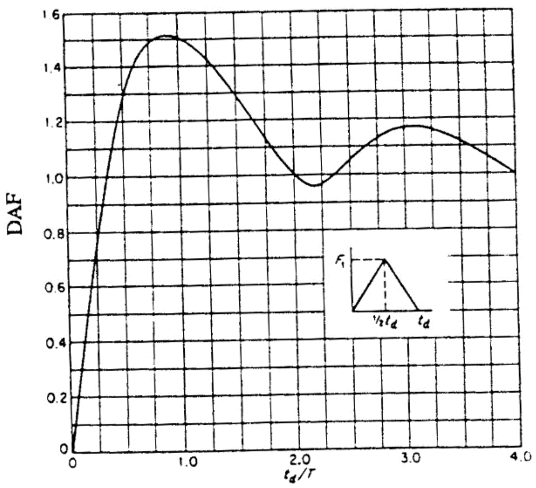
$$u_i(x) = \sin\frac{i\pi x}{L} \qquad M_{i\,eq} = \frac{mL}{2} \qquad K_{i\,eq} = \frac{(i\pi)^4 EI}{2L^3}$$

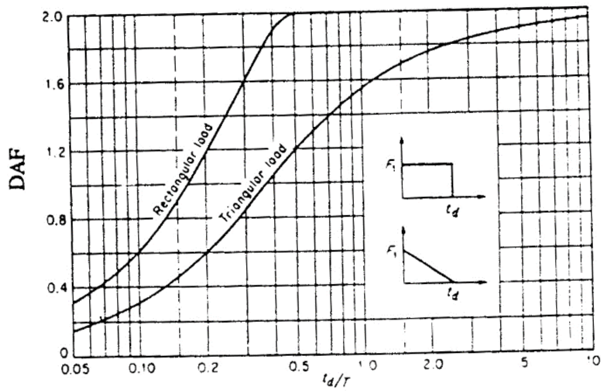
Ground motion participation factor

$$\Gamma = \frac{\int m\bar{u}dx}{\int m\bar{u}^2 dx}$$

Dynamic amplification factors







Numerical Integration Schemes:

Central Difference Method:

Acceleration $\ddot{u}_t = \frac{1}{\Delta t^2}\,(u_{t-\Delta t} - 2\,u_t + u_{t+\Delta t})$

Velocity $\dot{u}_t = \frac{1}{2\,\Delta t}\,(-u_{t-\Delta t} + u_{t+\Delta t})$

Linear Acceleration Method:

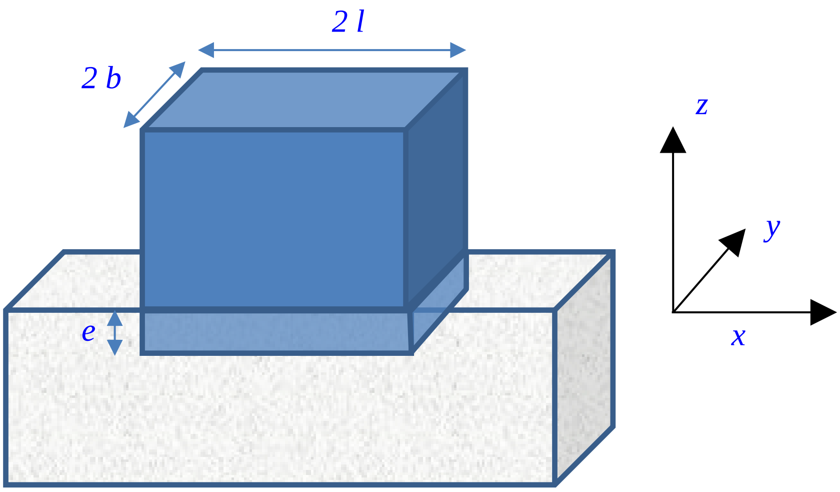
Acceleration $\ddot{u}(\tau) = \ddot{u}_n + (\ddot{u}_{n+1} - \ddot{u}_n)\,\frac{\tau}{\Delta t}$

Velocity $\dot{u}(\tau) = \dot{u}_n + \ddot{u}_n\tau + (\ddot{u}_{n+1} - \ddot{u}_n)\frac{\tau^2}{2\,\Delta t}$

Displacement $u(\tau) = u_n + \dot{u}_n\tau + \ddot{u}_n\frac{\tau^2}{2} + (\ddot{u}_{n+1} - \ddot{u}_n)\frac{\tau^3}{6\,\Delta t}$

Wolf Formulae:

Approximate relations for evaluating soil stiffness for an embedded prismatic structure of dimensions *2l* and *2b*, embedded to a depth *e* are:



$$K_{hx} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 2.4\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{hy} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 0.8\,\frac{l}{b} + 1.6\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_v = \frac{Gb}{2-\nu}\left[3.1\left(\frac{l}{b}\right)^{0.75} + 1.6\right]\left[1 + \left(0.25 + \frac{0.25b}{l}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{rx} = \frac{Gb^3}{1-\nu}\left[3.2\frac{l}{b} + 0.8\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35 + \frac{l}{b}}\left(\frac{e}{b}\right)^2\right)\right]$$

$$K_{ry} = \frac{Gb^3}{1-\nu}\left[3.73\left(\frac{l}{b}\right)^{2.4} + 0.27\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35 + \left(\frac{l}{b}\right)^4}\left(\frac{e}{b}\right)^2\right)\right]$$

$$K_{tor} = Gb^3\left[4.25\left(\frac{l}{b}\right)^{2.45} + 4.06\right]\left[\left(1 + \left(1.3 + 1.32\frac{b}{l}\right)\left(\frac{e}{b}\right)^{0.9}\right)\right]$$

Unit weight of soil:

$$\gamma = \frac{(G_s + eS_r)\gamma_w}{1+e}$$

where e is the void ratio, S_r is the degree of saturation, G_s is the specific gravity of soil particles.

For dry soil this reduces to

$$\gamma_d = \frac{G_s\gamma_w}{1+e}$$

Effective mean confining stress

$$p' = \sigma'_v \frac{(1+2K_o)}{3}$$

where σ'_v is the effective vertical stress, K_o is the coefficient of earth pressure at rest given in terms of Poisson's ratio ν as

$$K_o = \frac{\nu}{1-\nu}$$

Effective stress Principle:

$$p' = p - u$$

Shear modulus of sandy soils can be calculated using the approximate relation:

$$G_{\max} = 100 \frac{(3 - e)^2}{(1 + e)} (p')^{0.5}$$

where p' is the effective mean confining pressure in **MPa,** e is the void ratio and G_{max} is the small strain shear modulus in **MPa**

Shear modulus correction for strain may be carried out using the following expressions;

$$\frac{G}{G_{\max}} = \frac{1}{1 + \gamma_h}$$

where

$$\gamma_h = \frac{\gamma}{\gamma_r}\left[1 + a.\, e^{-b\left(\frac{\gamma}{\gamma_r}\right)}\right]$$

'a' and 'b' are constants depending on soil type; for sandy soil deposits we can take

$$a = -0.2 \ln N$$

$$b = 0.16$$

where N is the number of cycles in the earthquake, γ is the shear strain mobilised during the earthquake and γ_r is reference shear strain given by

$$\gamma_r = \frac{\tau_{\max}}{G_{\max}}$$

where

$$\tau_{\max} = \left[\left(\frac{1 + K_o}{2}\sigma_v' \sin\phi'\right)^2 - \left(\frac{1 - K_o}{2}\sigma_v'\right)^2\right]^{0.5}$$

Shear Modulus is also related to the shear wave velocity $\boldsymbol{v_s}$ as follows;

$$v_s = \sqrt{\frac{G}{\rho}}$$

where G is the shear modulus and ρ is the mass density of the soil.

Natural frequency of a horizontal soil layer f_n is;

$$f_n = \frac{v_s}{4H}$$

where v_s is shear wave velocity and H is the thickness of the soil layer.

SPGM
January, 2018