EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 5 May 2015   2 to 3.30

Module 4F1

CONTROL SYSTEM DESIGN

Answer not more than ***two*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4F1 Formulae sheet (3 pages)
Supplementary pages: two extra copies of Fig. 1 (Question 3)
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) An uncertain system is modelled as

$$G(s) = G_0(s)(1+\Delta(s)) \qquad (1)$$

where $G_0(s)$ is a known transfer function and $\Delta(s)$ is assumed only to be stable and to satisfy a bound $|\Delta(j\omega)| < h(\omega)$ for all ω. Let $K(s)$ stabilise $G_0(s)$ with unity gain negative feedback. Show that a necessary and sufficient condition for $K(s)$ to stabilise $G(s)$ is that

$$\left| h(\omega) \left(\frac{G_0(j\omega)K(j\omega)}{1+G_0(j\omega)K(j\omega)} \right) \right| \leq 1 \qquad (2)$$

for all ω. State clearly any results you use. [15%]

(b) Sketch the root-locus diagram for the system with transfer function [20%]

$$L_0(s) = \frac{s}{(s-1)^2}.$$

(c) Consider an unstable plant with the transfer function:

$$G_0(s) = \frac{s}{s^2-1}.$$

(i) Explain why this plant can only be stabilised with an unstable controller $K(s)$. [20%]

(ii) Explain why a stabilising controller $K(s)$ can be chosen so that the return ratio $L(s) = G_0(s)K(s)$ is equal to $kL_0(s)$ for $L_0(s)$ as in Part (b). Find the value of k so that the closed-loop system is stable with coincident poles. [20%]

(iii) Suppose the plant is subject to an uncertainty in the form of (1) with

$$\Delta(s) = \frac{\delta(s+1)}{s+9}$$

where δ is real but unknown. Find the greatest upper bound on $|\delta|$ guaranteed by Part (a) for which the controller of Part (c)(ii) stabilises $G(s)$. [Hint: you may find it helpful to find the maximum of the left hand side of (2) using a Bode diagram.] [25%]

2 (a) Explain the importance of the sensitivity function in feedback control system design. [20%]

(b) In a position control problem for a mechanical instrument the transfer function from actuator input to measured position of the end effector takes the form

$$G(s) = \frac{1}{ms^2 + cs + k} W(s)$$

where m, c, $k > 0$, and $W(s) = W_i(s)$ for $i = 1, \ldots, 4$, depending on the choice of sensor, according to the following possibilities:

$$W_1(s) = \frac{s}{s+1}, \quad W_2(s) = \frac{1-s}{s+1}, \quad W_3(s) = Ts + 1 \ \ (T > 0), \quad W_4(s) = 1.$$

It is desired to design a controller $K(s)$ to achieve the following specifications on the sensitivity function:

A: $|S(j\omega)| \leq \varepsilon$ for $0 \leq \omega \leq 1$;

B: $|S(j\omega)| \leq 1 + \delta$ for $1 \leq \omega < \infty$.

where $0 < \varepsilon < 1$ and $\delta \geq 0$.

(i) For $W(s) = W_1(s)$ show that specification A is not achievable for any $\varepsilon < 1$. [15%]

(ii) For $W(s) = W_2(s)$ and $\delta = 0.5$ find a positive lower bound on ε. [30%]

(iii) For $W(s) = W_3(s)$, by considering an appropriate return ratio or otherwise, show that the specifications are achievable with $\delta = 0$ and any positive $\varepsilon < 1$. [20%]

(iv) For $W(s) = W_4(s)$, are the specifications achievable with $\delta = 0$ for some choice of $\varepsilon < 1$? Justify your answer. [15%]

3 Figure 1 is the Bode diagram of a system with transfer function $G(s)$ for which a compensator $K(s)$ is to be designed. It may be assumed that $G(s)$ is a real-rational transfer function.

(a) (i) Sketch on a copy of Fig. 1 the expected phase of $G(j\omega)$ if $G(s)$ were stable and minimum phase. [15%]

(ii) Determine whether $G(s)$ has any right half plane poles or any right half plane zeros (it doesn't have both). Estimate their location (if there are any). [15%]

(iii) Comment briefly on any limitations that may be experienced in the design of $K(s)$. [10%]

(b) If a constant controller $K(s) = k$ is used, estimate the range of values of k for which the closed-loop system is internally stable. (Justify your answer carefully using the Nyquist stability criterion.) [20%]

(c) A feedback compensator $K(s)$ is sought to satisfy the following specifications:

A: Internal stability of the closed-loop;

B: $|G(j\omega)K(j\omega)| \geq 10$ for $\omega \leq 0.8$ rad s^{-1};

C: $|G(j\omega)K(j\omega)| \leq 1$ for $\omega \geq 20$ rad s^{-1};

D: Phase margin of at least 45°.

(i) Explain why the specfications cannot be met using a compensator with one pole and one zero. [Hint: you may find it helpful to consider the case of a lead compensator and a lag compensator separately.] [15%]

(ii) Design a compensator to satisfy the specifications. Sketch the Bode diagram of $K(s)$ and $G(s)K(s)$ for your design on a further copy of Fig. 1. [25%]

Two copies of Fig. 1 are provided on separate sheets. These should be handed in with your answers.

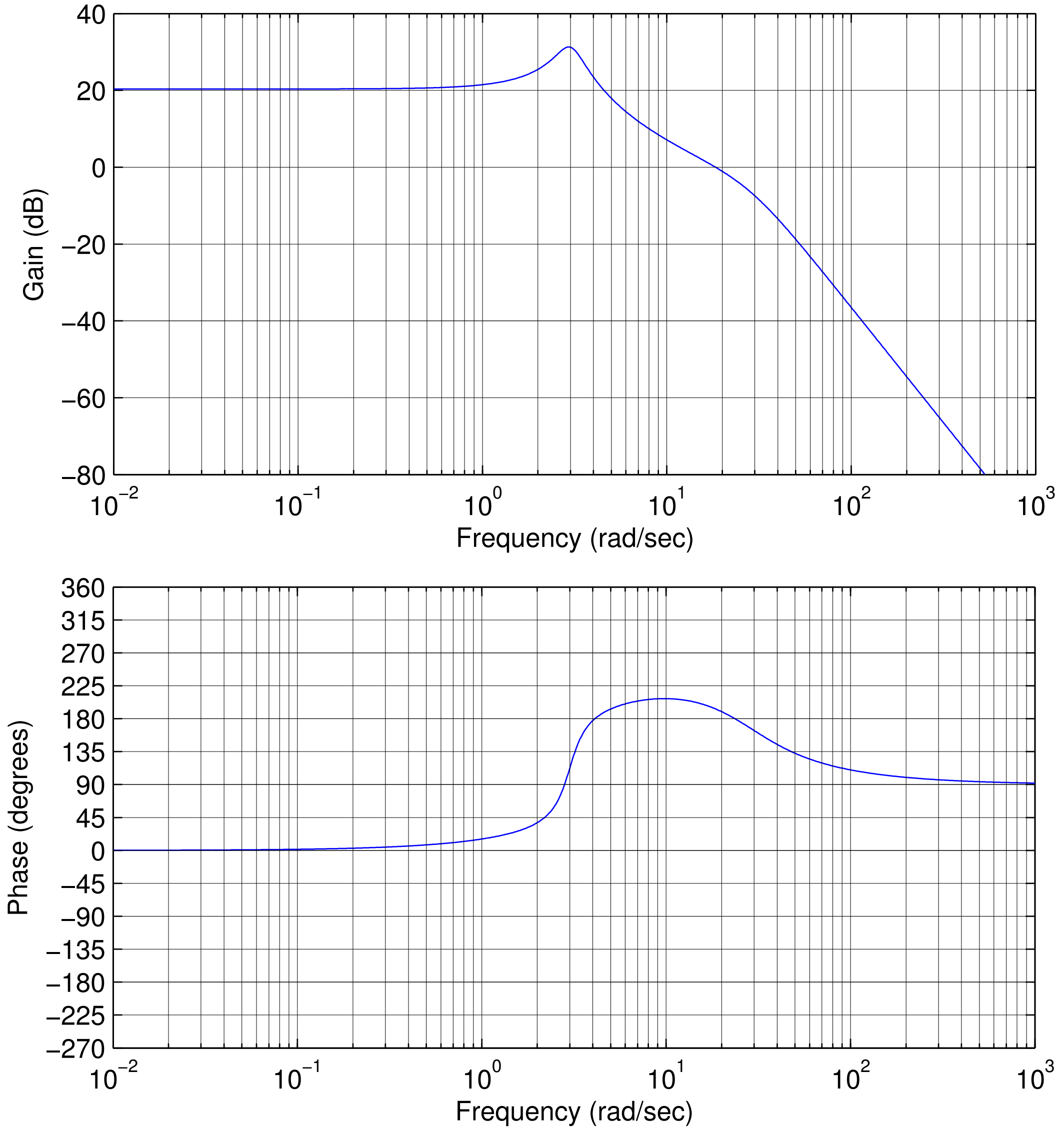

Fig. 1

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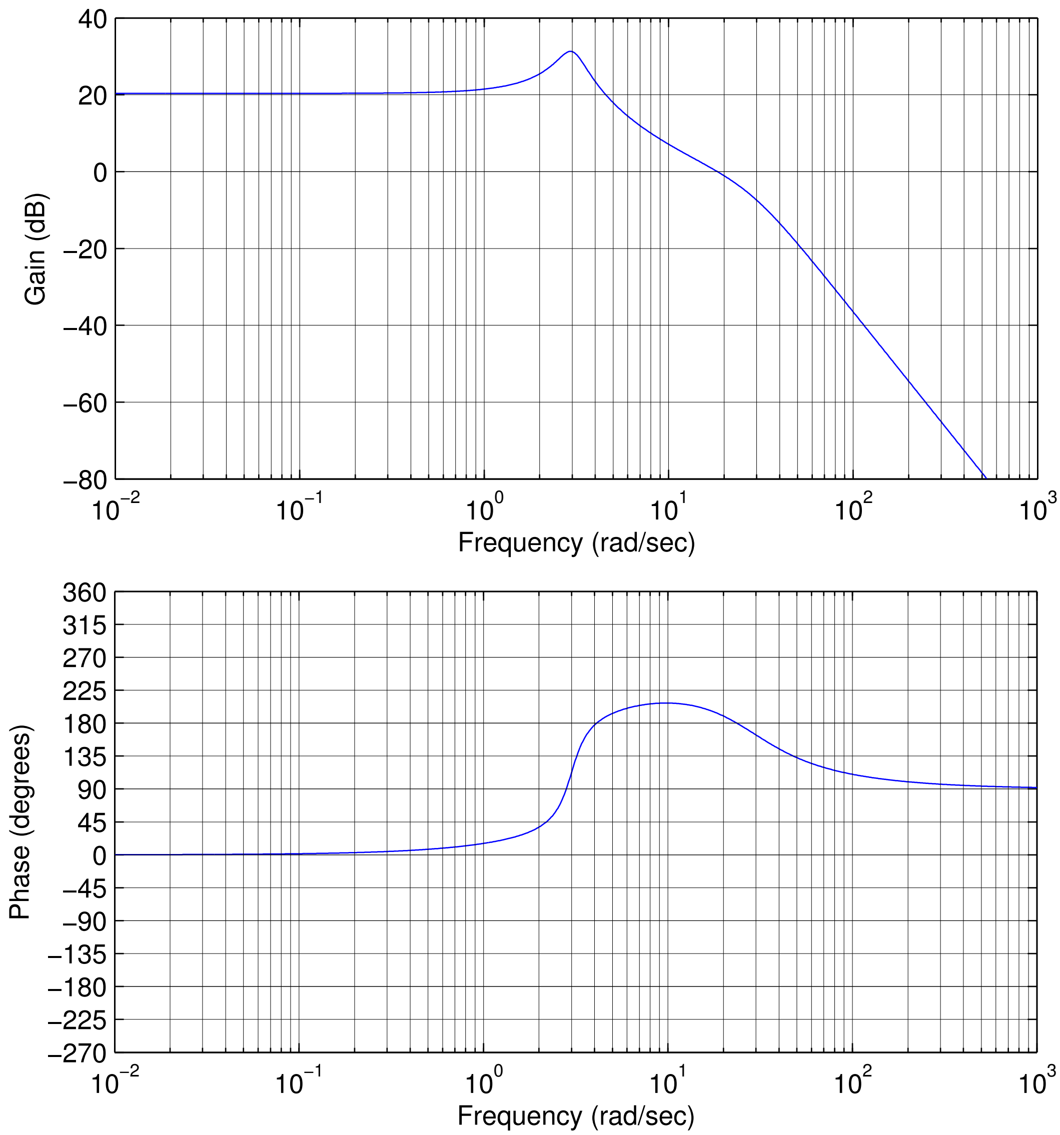

Extra copy of Fig. 1: Bode diagram for Question 3.

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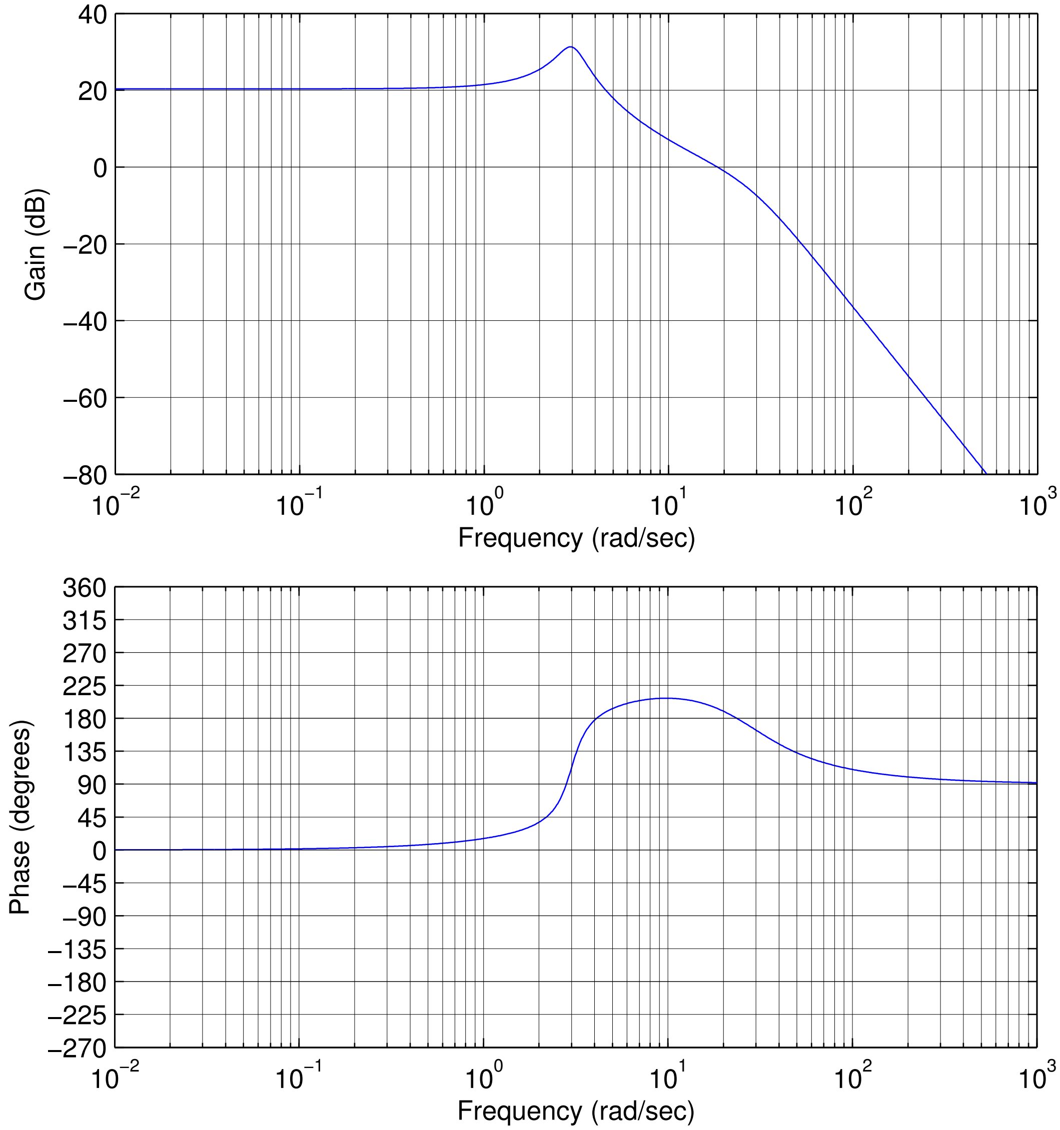

Extra copy of Fig. 1: Bode diagram for Question 3.

Engineering Tripos Part IIB

2015

Paper 4F1: Control System Design

Answers

1. (c)(ii) $k = 4$.
   (c)(iii) Greatest upper bound on $|\delta|$ is $\frac{5}{2}$.

2. (b)(ii) A positive lower bound on ϵ is $\frac{2}{3}$.
   (b)(iv) No.

3. (a)(ii) Two right half plane poles with approximate locations $0.6 \pm j3$.
   (b) Closed-loop stable for $0.07 < k < 1.6$ (approx).