Version MCS/4

EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 26 April 2016 2 to 3.30

Module 4F1

CONTROL SYSTEM DESIGN

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4F1 Formulae sheet (3 pages)
Supplementary pages:
one extra copy of Fig. 1 (Question 1)
two extra copies of Fig. 2 (Question 2)
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A control system is to be designed for the lateral motion of a vectored thrust aircraft. The transfer function for the linearised dynamics from lateral thrust F to lateral velocity v takes the form

$$G(s) = \frac{Js^2 - mgr}{Js^2(ms+c)}$$

where m is the mass of the aircraft, J is the roll moment moment of inertia, r is the distance of the thrusters below the centre of gravity, c is a damping coefficient and g is the acceleration due to gravity.

(a) Show that the sensitivity function $S(s)$ for the control system satisfies:

$$\int_0^\infty \frac{\sigma}{\sigma^2 + \omega^2} \log |S(j\omega)| \, d\omega = 0$$

where $\sigma = \sqrt{mgr/J}$. [20%]

(b) It is desired to design a controller $K(s)$ to achieve the following specifications on the sensitivity function:

A: $|S(j\omega)| \leq \varepsilon$ for $0 \leq \omega \leq \omega_1$;

B: $|S(j\omega)| \leq 1+\delta$ for $\omega_1 \leq \omega < \infty$,

for some $\omega_1 > 0$ to be determined, where $0 < \varepsilon < 1$ and $\delta > 0$ are given. Show that it is necessary that

$$\omega_1 \leq \sigma \tan\left(\frac{\pi}{2} \, \frac{\log(1+\delta)}{\log((1+\delta)\varepsilon^{-1})}\right).$$

[25%]

(c) What advice should the control engineer provide on the choice of r? [10%]

(d) Figure 1 shows parts of the frequency response locus for the return ratio $L(s)$ for the plant $G(s)$ together with a compensator $K(s)$ which may be assumed to have no poles on the imaginary axis. Frequencies are indicated with linear spacing on each subplot: $\omega = 0.2, 0.3, \ldots, 1.0$ and $\omega = 4, 6, \ldots, 20$ rad s^{-1}. Sketch the complete Nyquist diagram of $L(s)$ and determine the closed-loop stability with the compensator $kK(s)$ for all $k > 0$. [20%]

(e) By use of a sketch on the extra copy of Figure 1 estimate the position of any closed-loop poles close to the imaginary axis for $k = 0.15$, $k = 1.5$ and $k = 2.5$. [25%]

An extra copy of Fig. 1 is provided on a separate sheet. This should be handed in with your answers.

(cont.

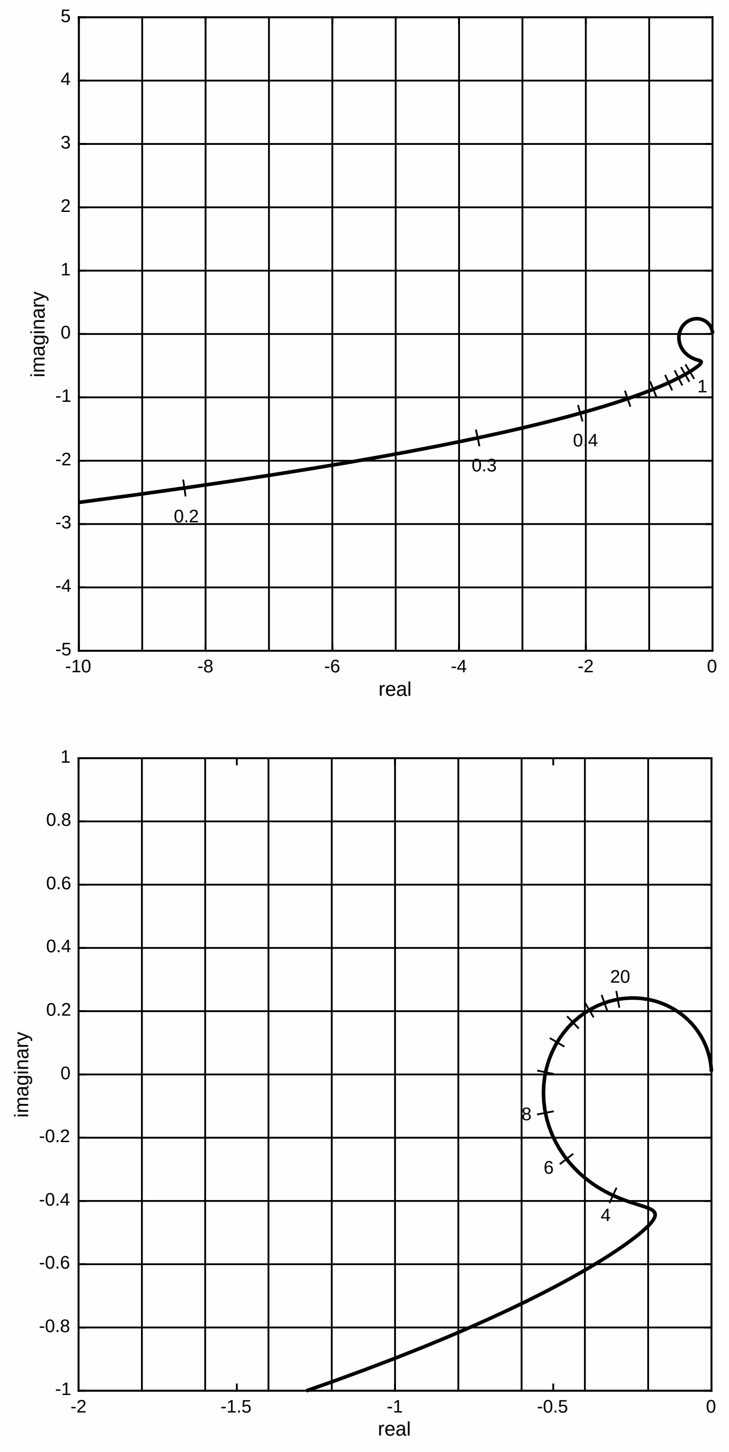

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Fig. 1

2 Figure 2 is the Bode diagram of a system with transfer function $G(s)$ for which a compensator $K(s)$ is to be designed. It may be assumed that $G(s)$ is a real-rational transfer function.

(a) (i) Sketch on a copy of Fig. 2 the expected phase of $G(j\omega)$ if $G(s)$ were stable and minimum phase. [10%]

(ii) Explain why your sketch only allows a *least* number of right half plane poles to be deduced. [10%]

(iii) Determine this least number of right half plane poles of $G(s)$ and the corresponding number of right half plane zeros. Estimate their location (if there are any). [10%]

(iv) Comment briefly on any limitations that may be experienced in the design of $K(s)$. [10%]

(b) You may assume that the number of right half plane poles of $G(s)$ equals the least number found in Part (a)(iii).

(i) If a constant controller $K(s) = k$ is used, estimate the range of values of k for which the closed-loop system is internally stable. (Justify your answer carefully using the Nyquist stability criterion.) [20%]

(ii) Design a feedback compensator $K(s)$ to satisfy the following specifications:

A: Internal stability of the closed-loop;

B: $|G(j\omega)K(j\omega)| \geq 10$ for $\omega \leq 0.1\ \mathrm{rad\,s^{-1}}$;

C: Phase margin of at least 45°.

Sketch the Bode diagram of $K(s)$ and $G(s)K(s)$ for your design on a further copy of Fig. 2. [Hint: you may find it useful to first design a compensator to satisfy A and C with some extra margin, and then to further modify it to satisfy B.] [40%]

Two copies of Fig. 2 are provided on separate sheets. These should be handed in with your answers.

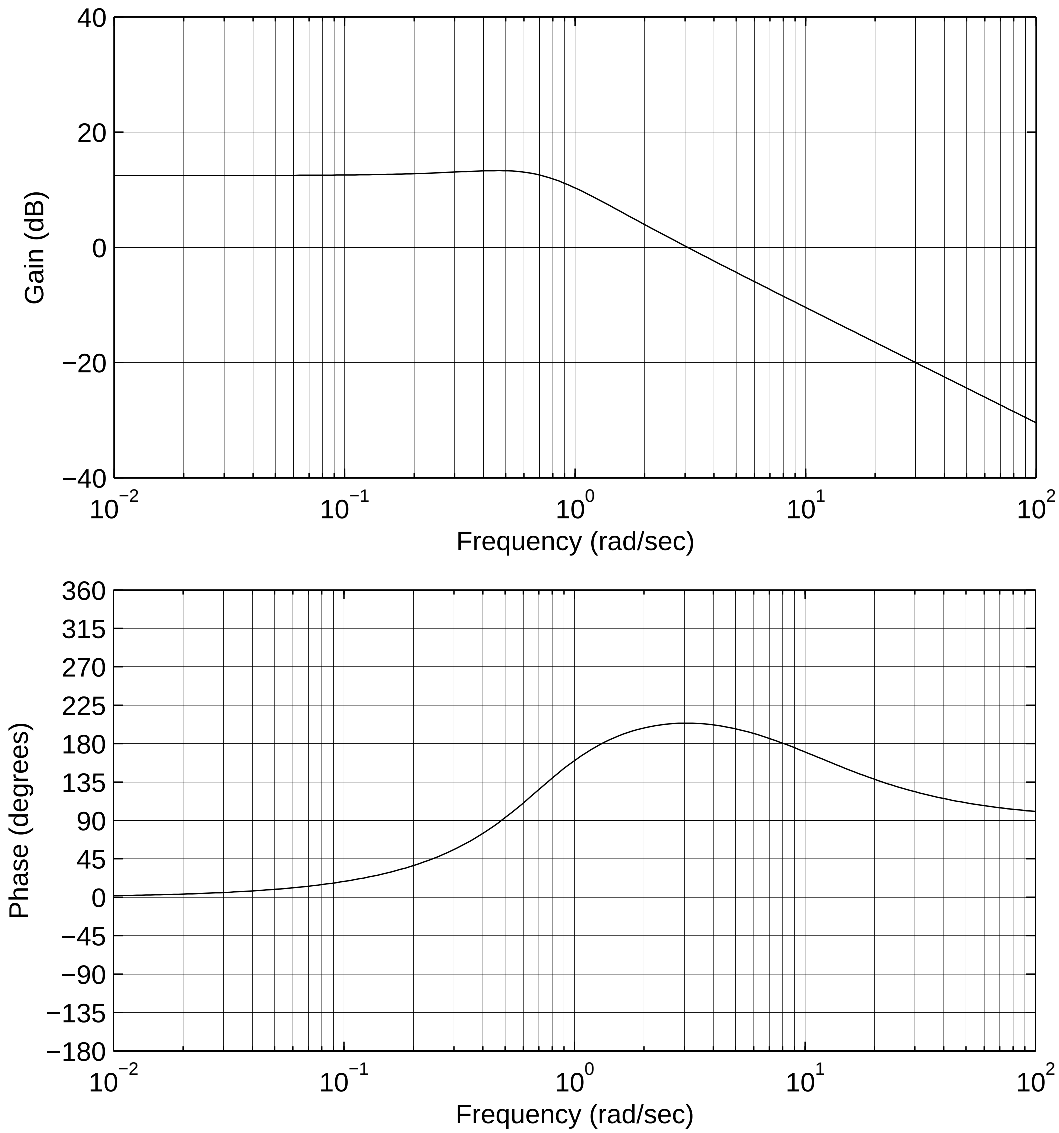

Fig. 2

(TURN OVER

3 A control system is to be designed for a lift. The open loop transfer function from motor drive u to lift vertical position y is modelled as

$$G(s) = \frac{as+1}{s^2}$$

where $a > 0$ is a constant.

(a) Sketch the root-locus diagram for positive gain k. [15%]

(b) Find the value of k for which the closed-loop system has repeated poles in the left half plane. [15%]

(c) For the value of k found in Part (b) find the closed-loop transfer function from reference input r to error signal $e = r - y$. [10%]

(d) If $r(t)$ is a unit step, show by considering the Laplace transform, or otherwise, that

$$\int_0^\infty e(t)\,dt = 0.$$

[15%]

(e) Without calculating the response exactly, sketch the form of $e(t)$ when $r(t)$ is a unit step. Explain why such a response is unsatisfactory for a lift control system. [15%]

(f) Explain why a control system could be designed so that the transfer function from r to y equals

$$\frac{1}{as+1}.$$

Why would this be more satisfactory? [15%]

(g) Design an internally stable control system for the lift to achieve the transfer function from r to y given in Part (f) . [15%]

END OF PAPER

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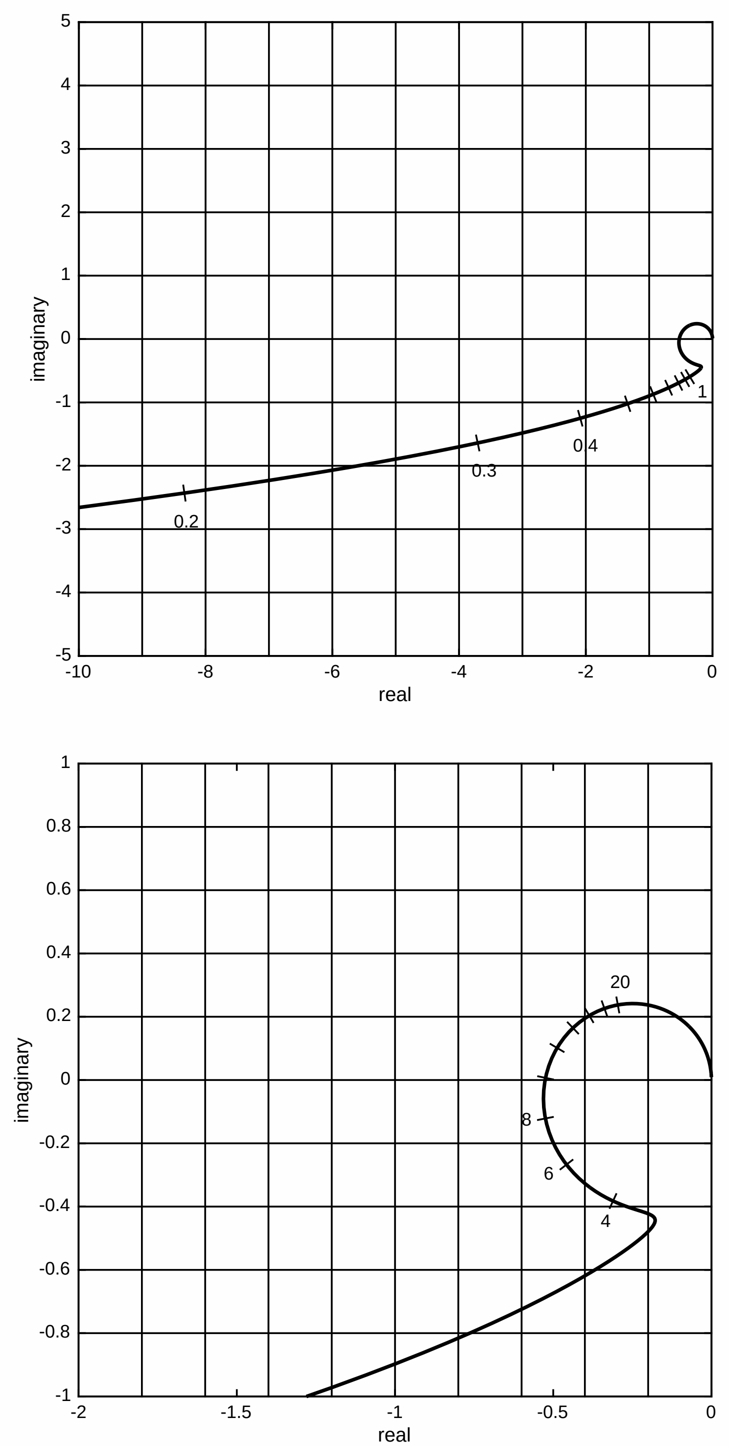

Extra copy of Fig. 1: Nyquist diagram for Question 1.

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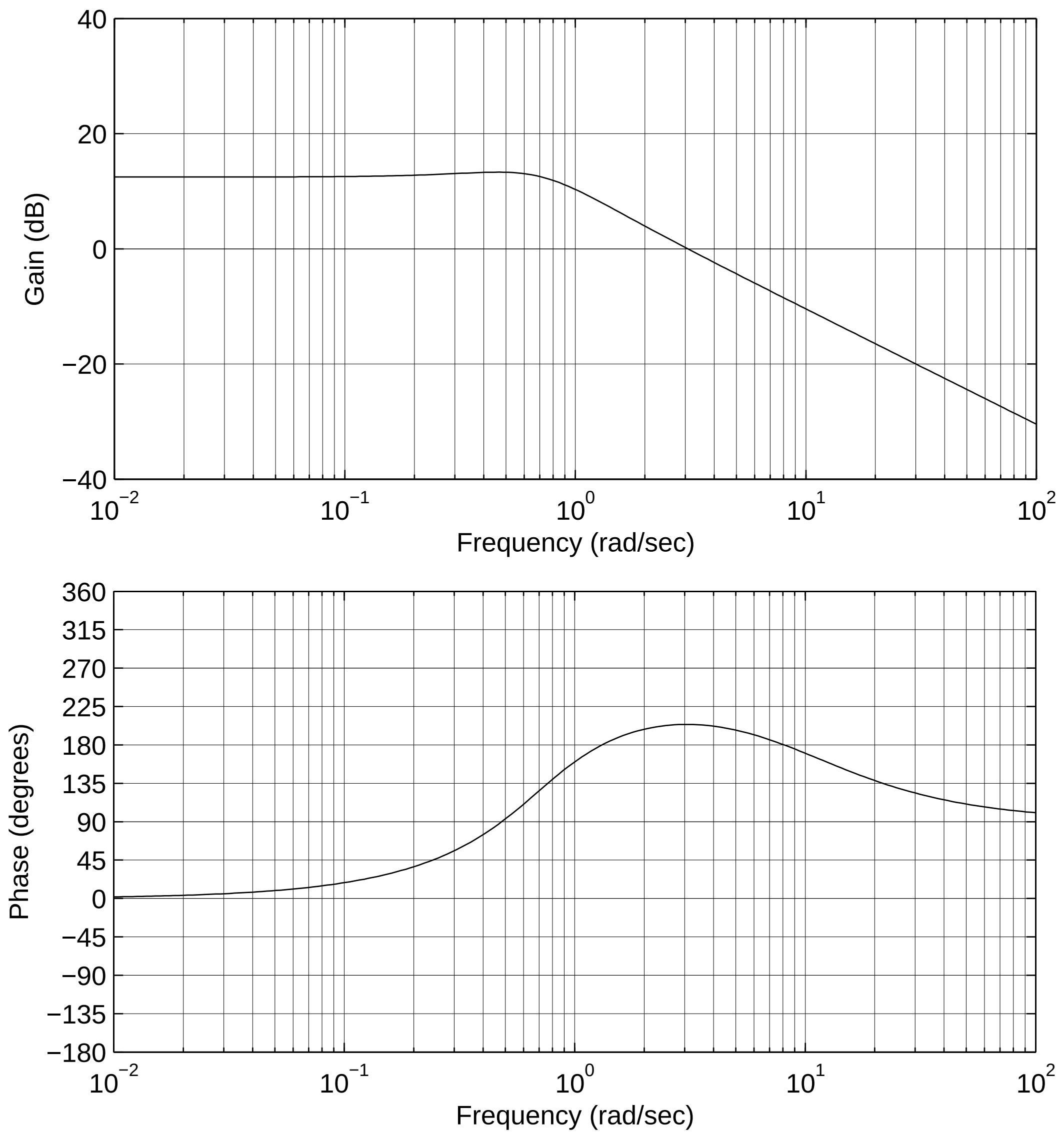

Extra copy of Fig. 2: Bode diagram for Question 2.

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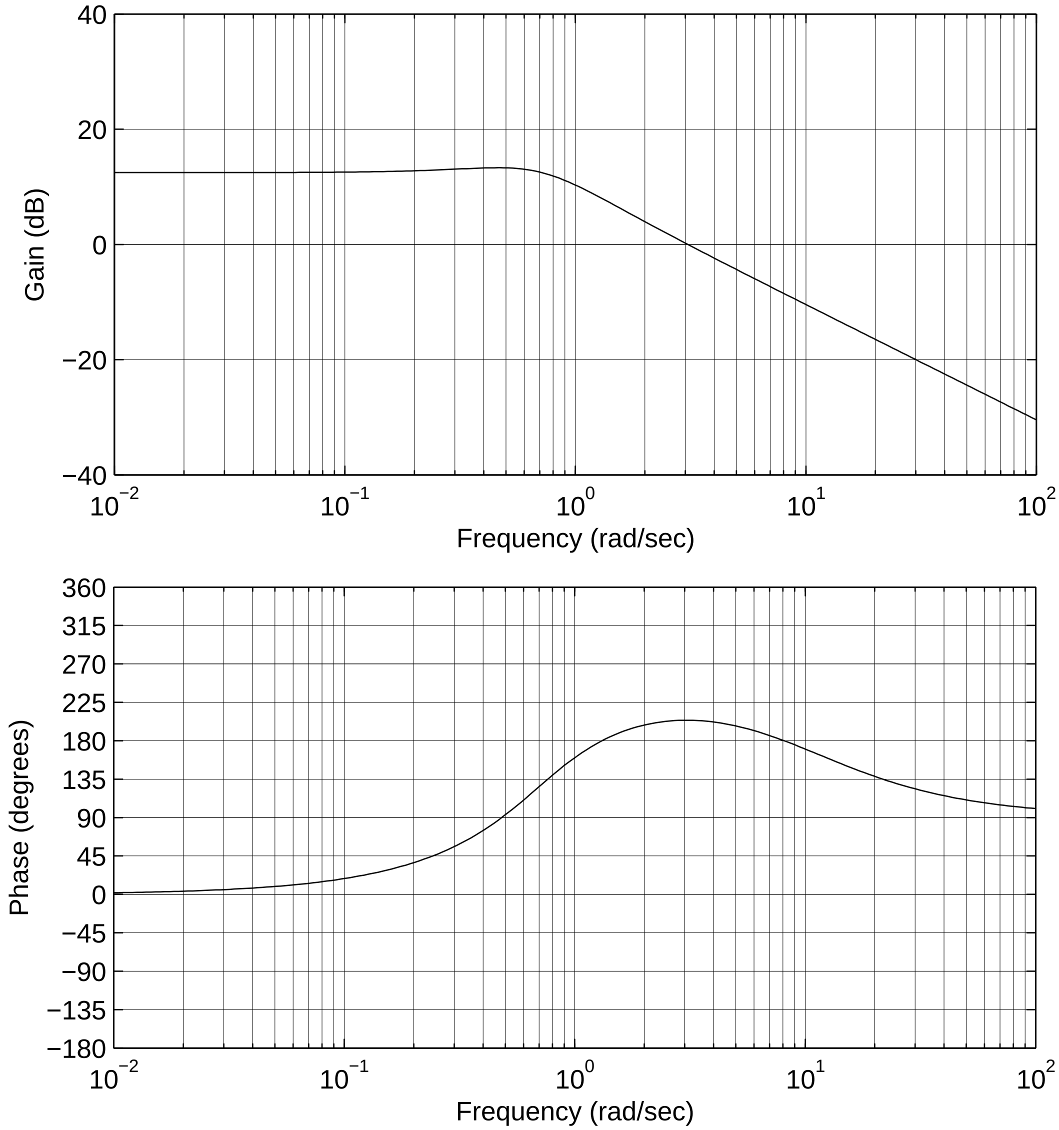

Extra copy of Fig. 2: Bode diagram for Question 2.