EGT3
ENGINEERING TRIPOS PART IIB

Thursday 25 April 2019 2 to 3.40

Module 4F1

CONTROL SYSTEM DESIGN

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4F1 Formulae sheet (3 pages)
Supplementary pages: two extra copies of Fig. 3 (Question 2)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) State carefully, but do not prove, the Small Gain Theorem for a system with stable real-rational transfer function $H(s)$. [15%]

(b) A control problem for a micro-mechanical system is modelled as in Fig. 1. It is desired to control the position $y(t)$ of the mass m from the position input $x(t)$ through a flexible spring element of stiffness k. The position of the mass cannot be directly measured but a sensor for the spring force $f(t) = k(x(t) - y(t))$ is available.

(i) Show that the transfer function $T_{\bar{x}\to\bar{f}}$ is given by [15%]

$$G(s) = \frac{kms^2}{ms^2 + k}.$$

(ii) Explain why this system cannot be stabilised by proportional feedback. [15%]

(iii) The actuator dynamics relating true position $x(t)$ to demanded position are modelled by a first-order lag:

$$G_a(s) = \frac{1}{\tau s + 1}.$$

Using the Routh-Hurwitz criterion, or otherwise, show that the system including actuator dynamics is stabilised by a proportional gain controller $K(s) = \alpha$ in the standard negative feedback configuration if and only if $\alpha > 0$. [15%]

(iv) The system is subject to uncertainty in the mass which is represented as $m = (1 + \Delta)m_0$ where m_0 is the nominal mass. Verify that the block diagram arrangement of Fig. 2 specifies the correct transfer function $T_{\bar{x}\to\bar{f}}$. [10%]

(v) It is desired to investigate the case where Δ is taken to be frequency dependent. Use the Small Gain Theorem to find a necessary and sufficient condition for the feedback system of Fig. 2 to be stable for all stable $\Delta(s)$ satisfying $|\Delta(j\omega)| < h$ for some constant $h > 0$. [15%]

(vi) Show that the condition of Part (b)(v) implies $h < 1$ when $K(s) = \alpha > 0$. Compare this robustness guarantee to the result of Part (b)(iii). [15%]

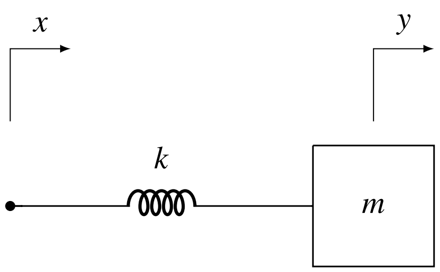

Fig. 1

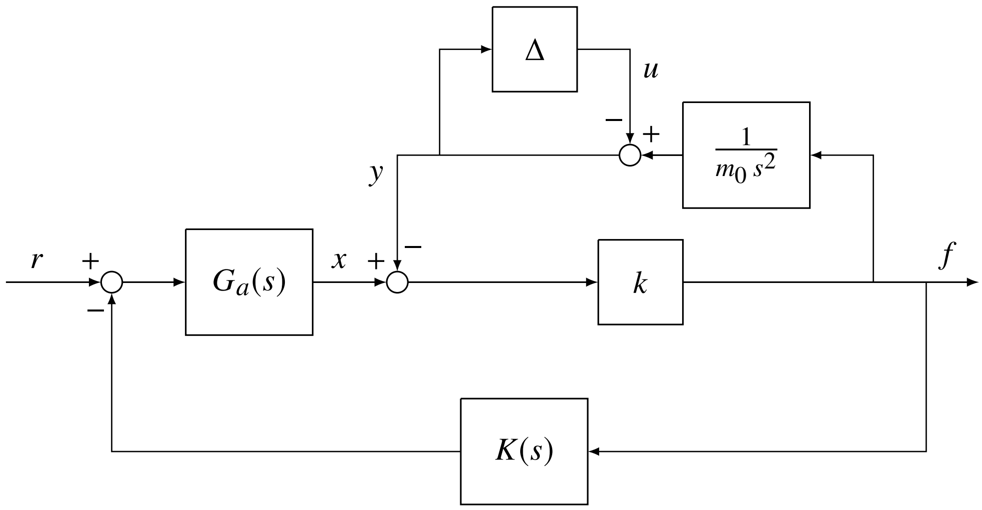

Fig. 2

2 Figure 3 is the Bode diagram of a system with transfer function $G(s)$ for which a compensator $K(s)$ is to be designed. It is known that the system has one pole satisfying $\mathrm{Re}(s) > 0$.

(a) (i) Sketch on a copy of Fig. 3 the phase plot of the stable and minimum phase transfer function with the same gain (magnitude) plot as that of G(s). [10%]

(ii) Estimate the location of the right half plane pole and suggest the form of any stable all-pass factor (if any). [10%]

(iii) Comment briefly on any limitations that may be experienced in the design of $K(s)$. [10%]

(iv) Determine the number of poles of $G(s)$ at the origin using the Bode magnitude plot. [10%]

(b) (i) Sketch the complete Nyquist diagram of $G(s)$, paying close attention to any required indentations of the Nyquist D-contour. [10%]

(ii) Explain carefully with reference to the Nyquist stability criterion why the closed-loop system will not be stable with a constant controller $K(s) = k$, for any k either positive or negative. [10%]

(c) (i) Find a $K(s) = kK_1(s)^2$, where $K_1(s)$ is a phase-lead compensator, for which the closed-loop system is stable with a phase margin of at least 45°. Show on another copy of Fig. 3 the effect of this compensation on the open-loop transfer function. [20%]

(ii) Explain carefully with reference to a new Nyquist diagram why the system is closed-loop stable. [10%]

(iii) Determine the smallest pure time delay which could make the system unstable when placed in series with the controller. [10%]

Two copies of Fig. 3 are provided on separate sheets. These should be handed in with your answers.

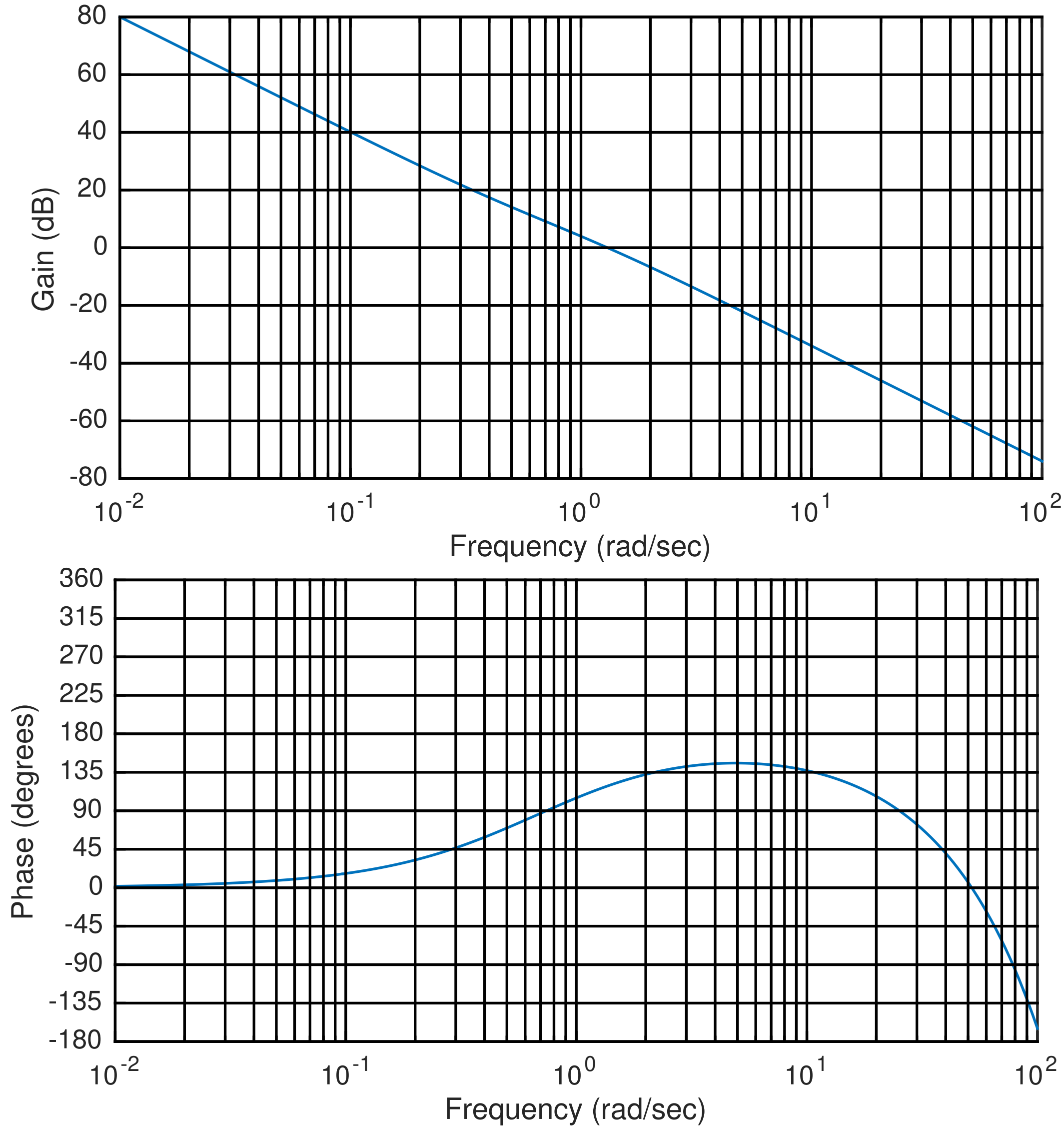

Fig. 3

(TURN OVER

3 (a) Consider a plant with transfer-function:

$$G(s) = \frac{(a_1 s + 1)(a_2 s + 1)\cdots(a_{n-1} s + 1)}{b_n s^n + \cdots + b_1 s + 1} = \frac{a(s)}{b(s)}$$

where $a_i \neq 0$ for all i, $b_n \neq 0$ and $b(s)$ has all its roots in $\mathrm{Re}(s) < 0$.

(i) Find an expression for the initial slope of the step response of the plant. Hence show that there is initial undershoot in the step response if and only if an odd number of the a_i are negative. [15%]

(ii) Suppose that $a_i < 0$ for some i. By considering the definition of the Laplace transform of the step response $y(t)$, or otherwise, show that [10%]

$$\int_0^\infty y(t) e^{t/a_i}\, dt = 0.$$

(iii) Deduce that, if one or more of the a_i is negative, then there will always be at least one $t_1 > 0$ where $y(t_1) = 0$. [10%]

(b) Let

$$G(s) = \frac{(1-s)^2}{s(s^2 + 2s + 12)}.$$

(i) Sketch the root-locus diagram of $G(s)$ for $k > 0$ and $k < 0$. [Hint: you may use the fact that $G'(s) = 0$ at the four points $s = -2, -1, 1, 6$ without verifying this.] [20%]

(ii) Determine the value of feedback gain k for which a closed-loop pole is at $s = -1$, and find the location of the other closed-loop poles at this gain. [15%]

(iii) Hence, or otherwise, design a two-degree-of-freedom control system for this plant for which the transfer-function $T_{\bar{r} \to \bar{y}}$ relating output to reference input is equal to

$$R(s) = \frac{(1-s)^2}{(s+1)^3}.$$

[15%]

(iv) Without attempting to calculate it, provide an approximate sketch of the step response of $R(s)$ which indicates the main features that may be expected. State what features of the step response are unavoidable in any allowed choice of $R(s)$ satisfying $R(0) = 1$. [15%]

END OF PAPER

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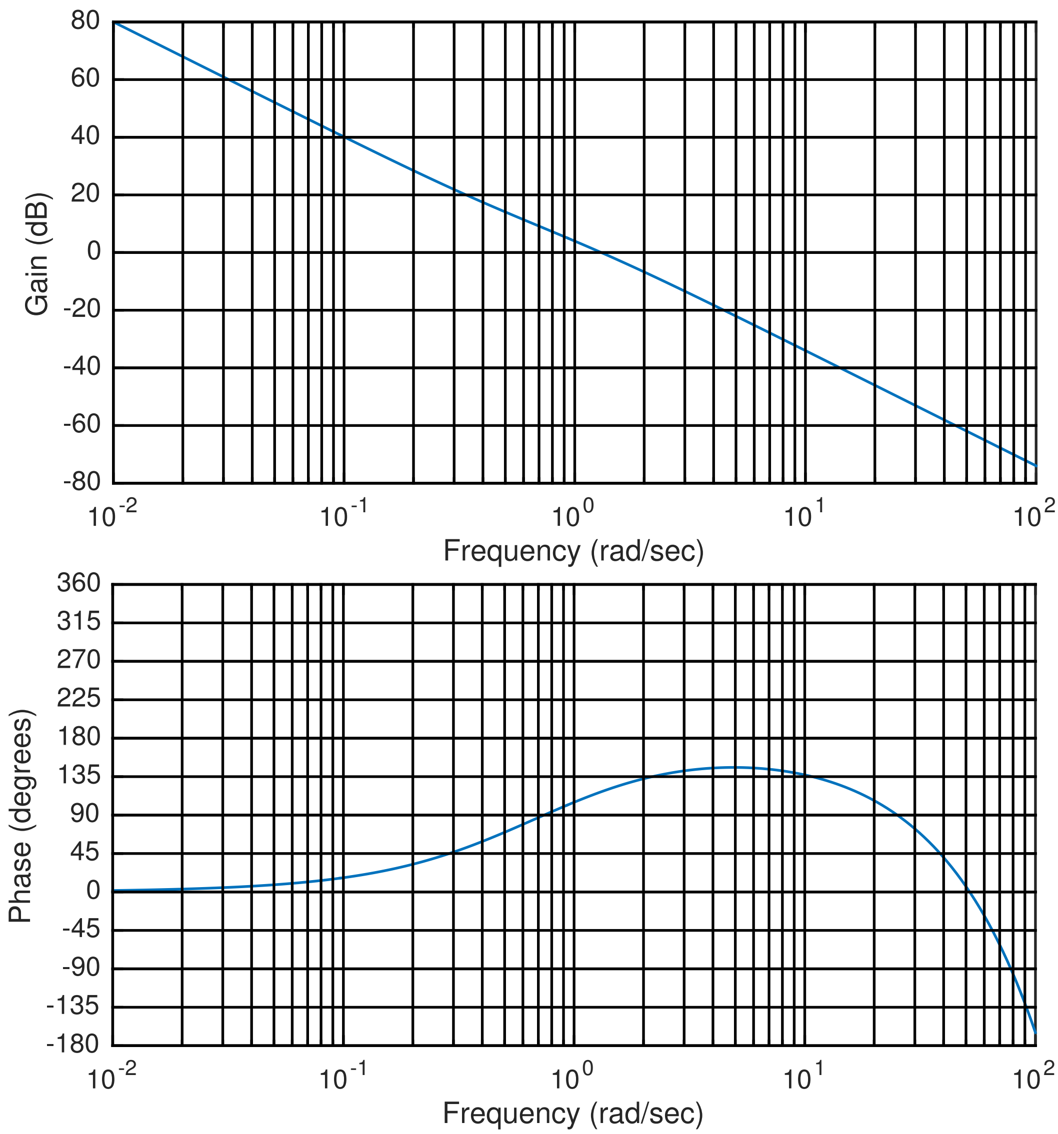
Thursday 25 April 2019, Module 4F1, Question 2.



Extra copy of Fig. 3: Bode diagram for Question 2.

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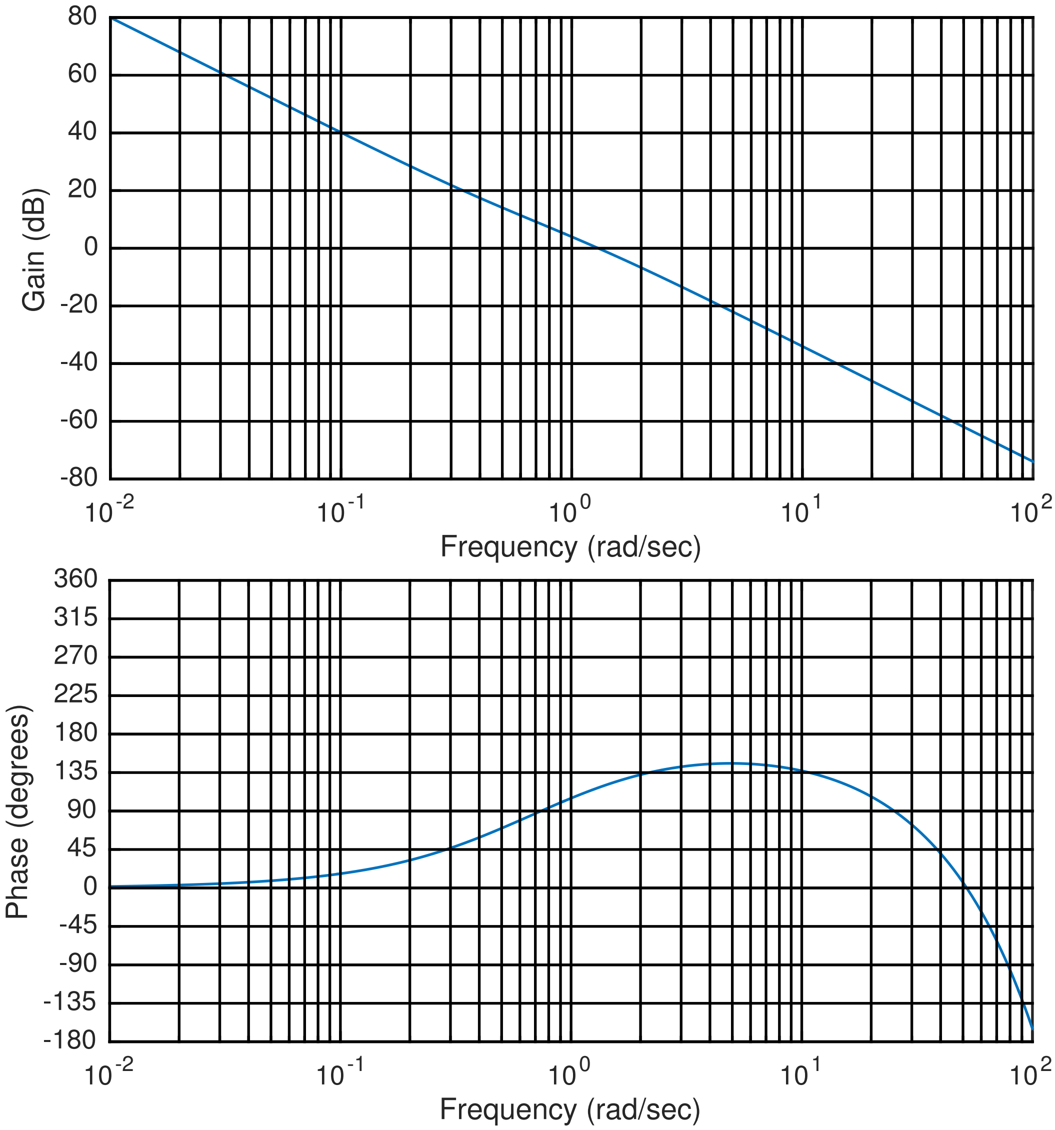

Extra copy of Fig. 3: Bode diagram for Question 2.

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Engineering Tripos Part IIB

2019

Paper 4F1: Control System Design

Answers

2. (b)(ii) pole ≈ 1. *Stable* all-pass factor $= e^{-sT}$, $T = 0.06$.

3. (a)(i)

$$\frac{a_1 a_2 \cdots a_{n-1}}{b_n}$$

(b)(ii) $k = 11/4$, poles at $-1, -1, -11/4$.