Version FF/3

EGT3
ENGINEERING TRIPOS PART IIB

Monday 30 April 2018   2.00 to 3.40

Module 4F2

ROBUST & NONLINEAR CONTROL

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Consider the closed loop system in Fig. 1 where G is the plant transfer function and K is the controller transfer function. Assume that both transfer functions are in $\mathscr{H}_\infty$.

(a) Using the block diagram in Fig. 1 show that

$$\begin{bmatrix} I & -K \\ -G & I \end{bmatrix}^{-1} = \begin{bmatrix} (I-KG)^{-1} & K(I-GK)^{-1} \\ G(I-KG)^{-1} & (I-GK)^{-1} \end{bmatrix}.$$

(Hint: if needed, use the identities $M(I-NM)^{-1} = (I-MN)^{-1}M$ and $(I-NM)^{-1} = I+N(I-MN)^{-1}M$, where M and N are two generic matrices.) [15%]

(b) Show that if $\| KG \|_\infty < 1$ then $\begin{bmatrix} I & -K \\ -G & I \end{bmatrix}^{-1}$ is in $\mathscr{H}_\infty$. Is the closed loop system internally stable for $\| KG \|_\infty < 1$? Explain your answer. [25%]

(c) Consider the Bode diagrams in Fig. 2 representing the magnitude of plant G and of three controllers K denoted respectively by K_1, K_2 and K_3. Which controllers guarantee nominal closed loop stability? Explain your answer. [20%]

(d) Using the small gain theorem, derive the condition for robust stability of the closed loop system in Fig. 1 in the presence of an additive uncertainty Δ on G. [15%]

(e) Consider any additive uncertainty Δ on G such that $\| \Delta \|_\infty < b$. For each of the stabilizing controllers K_i found in (c), use the small gain theorem to find the largest b such that robust stability is guaranteed. [25%]

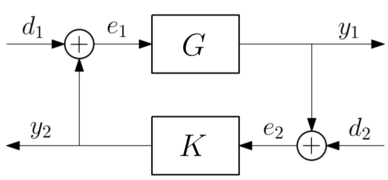

Fig. 1

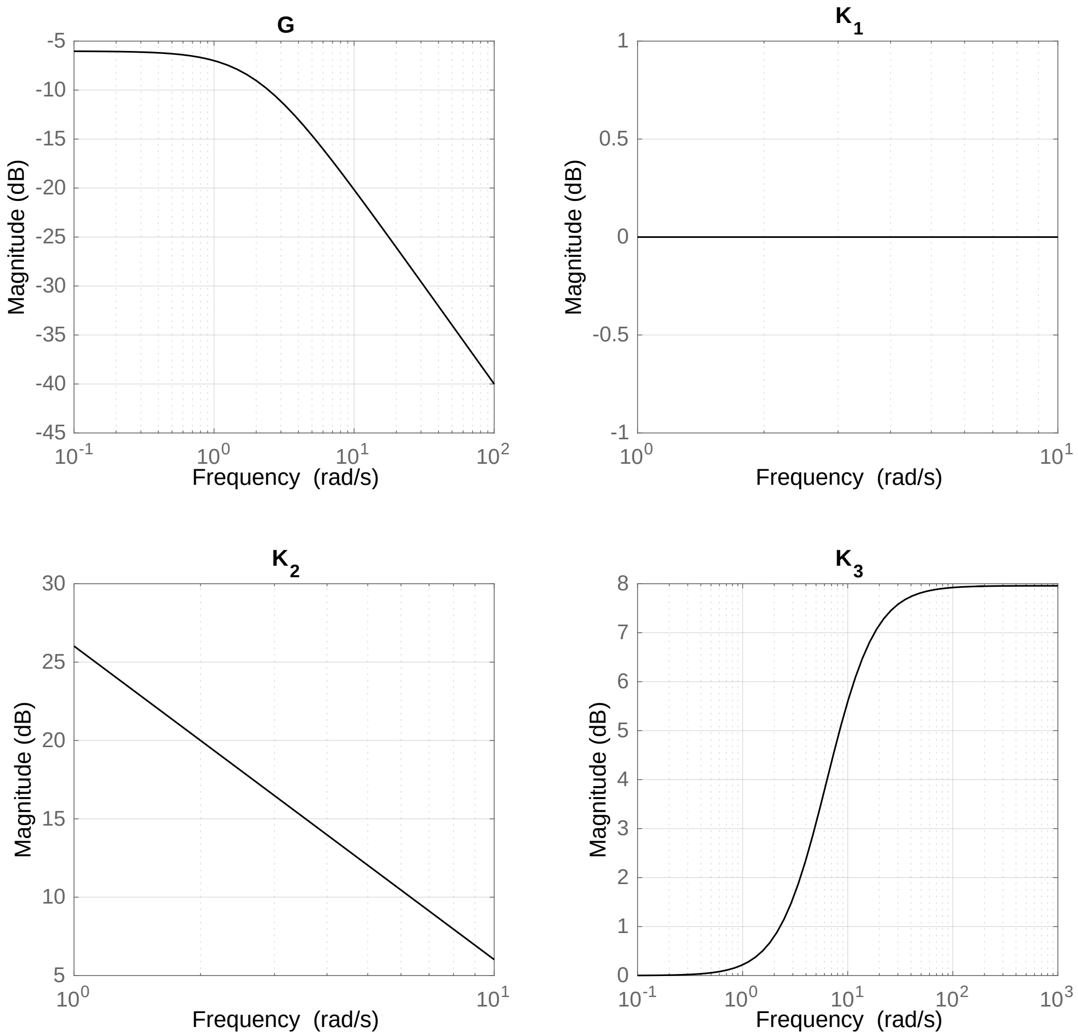

Fig. 2

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2 Consider the closed loop system in Fig. 3 defined by controller K, actuator A, and plant P. The plant transfer function is $P(s) = \frac{1}{s}$. The actuator model is $A(s) = A_0(s)(1+\bar{\Delta}(s))$, which consists of a nominal transfer function $A_0(s) = \frac{1}{s+2}$ and a multiplicative uncertainty on the input $\bar{\Delta}(s)$. $K(s) = k > 0$ is a proportional controller.

(a) Suppose that $A(s) = \frac{1}{s+a}$ where $1 < a < 2$. Show that $\bar{\Delta}(s) = \Delta(s)W(s)$ for $W(s) = \frac{2}{s+2}$ and $\| \Delta(s) \|_\infty < 1$. Draw a new block diagram representing the closed loop system with multiplicative uncertainties on the actuator. [20%]

(b) Show that any control gain $k > 0$ guarantees that the nominal transfer functions $T_{r \to e}(s)$, from the reference r to the error e, and $T_{d \to y}(s)$, from the disturbance d to the output y, are in $\mathscr{H}_\infty$. [20%]

(c) Consider the multiplicative uncertainty in part (a). Suppose that the nominal closed loop system is internally stable. Using the small gain theorem, derive the range of positive gains k that guarantee robust stability. [20%]

(d) Consider any frequency $\omega > 0$ rad/s. Show that, for a sufficiently large k, nominal tracking improves at frequency ω. What is the effect of large gains k on the robustness of the system to multiplicative uncertainties on the plant output? Explain your answer. [20%]

(e) Define the robust performance problem. Introduce the notion of generalized plant and briefly discuss a technique to address the robust performance problem. [20%]

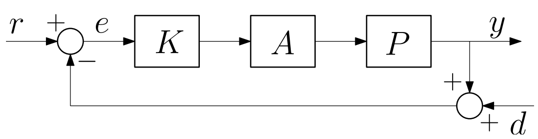

Fig. 3

3 (a) Explain what is meant by the following for a dynamical system with state space representation $\dot{x} = f(x)$:

(i) Stable equilibrium point;

(ii) Asymptotically stable equilibrium point;

(iii) Globally asymptotically stable equilibrium point;

(iv) Domain of attraction of an equilibrium point.

[20%]

(b) Describe Lyapunov's indirect method for deducing stability properties of an equilibrium point of a nonlinear system via its linearization. Explain also the limitations of this method. [20%]

(c) Consider the system

$$\dot{x}_i = -\alpha_i x_i - \sum_{j=1}^{m} f_j(y_j), \quad i = 1, \ldots, n$$

$$\dot{y}_j = -\beta_j y_j + \sum_{i=1}^{n} g_i(x_i), \quad j = 1, \ldots, m$$

where for all i, j we have that α_i, β_j are constants that satisfy $\alpha_i \geq 0$ and $\beta_j > 0$, f_j and g_i are Lipschitz continuous functions that are non-decreasing, $f_j(0) = 0$, $f_j(y_j)y_j > 0$ for $y_j \neq 0$, $g_i(0) = 0$, $g_i(x_i)x_i > 0$ for $x_i \neq 0$.

(i) Using the function

$$V(x,y) = \sum_{i=1}^{n} \int_0^{x_i} g_i(z)dz + \sum_{j=1}^{m} \int_0^{y_j} f_j(z)dz$$

show that the origin is an asymptotically stable equilibrium if it is the only equilibrium point of the system. [45%]

(ii) Is the origin also globally asymptotically stable? Provide a justification to your answer. [15%]

4 (a) Find the describing function for the nonlinearity given by

$$f(e) = \begin{cases} 0 & |e| \leq \delta, \\ e & |e| > \delta \end{cases}$$

where $\delta \geq 0$ is a constant. [40%]

(b) The nonlinearity $f(e)$ is connected in negative feedback with a linear system with transfer function

$$G(s) = \frac{\alpha}{(2s+1)^2}$$

where $\alpha > 0$ is a constant. Use the circle criterion to find the values of $\alpha > 0$ for which stability of the feedback system is guaranteed. Discuss if the circle criterion implies that the feedback system will be unstable when α takes values outside this range. [20%]

(c) Using the fact that the describing function derived in part (a) is less than or equal to 1, explain whether the describing function method predicts the existence of a limit cycle for the feedback system described in (b). [20%]

(d) Consider now the negative feedback interconnection of $f(e)$ with a linear system with transfer function

$$G(s) = \frac{\beta}{(s+1)^3}$$

where $\beta > 0$ is a constant. Also let $\delta = 0$ in $f(e)$. Find the value of $\beta > 0$ for which the feedback system has a steady-state oscillation. [20%]

END OF PAPER