EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 30 April 2019 2 to 3.40

Module 4F2

ROBUST AND NONLINEAR SYSTEMS AND CONTROL

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A feedback system has reference input $\hat{r}(s)$, disturbance $\hat{d}(s)$, error $\hat{e}(s)$, plant input $\hat{u}(s)$ and plant output $\hat{y}(s)$ related by: $\hat{e}(s) = \hat{r}(s) - \hat{y}(s)$, $\hat{y}(s) = G(s)\hat{u}(s) + \hat{d}(s)$, $\hat{u}(s) = K(s)\hat{e}(s)$, where $G(s)$ and $K(s)$ are the plant and controller transfer function matrices, respectively, and ˆ denotes the Laplace transform. Define $L(s) = G(s)K(s)$.

(a) (i) Sketch a block diagram of the feedback system and show that

$$\hat{y}(s) = S(s)\hat{d}(s) + T(s)\hat{r}(s)$$

where $S(s) = (1 + L(s))^{-1}$ and $T(s) = (1 + L(s))^{-1}L(s)$. [15%]

(ii) Show that

$$\underline{\sigma}(T(j\omega)) \geq 1 - \overline{\sigma}(S(j\omega)), \quad (1)$$

and

$$\overline{\sigma}(S(j\omega)) \leq \frac{1}{\underline{\sigma}(L(j\omega)) - 1} \quad (2)$$

when $\underline{\sigma}(L(j\omega)) > 1$, where $\overline{\sigma}(\cdot)$ and $\underline{\sigma}(\cdot)$ denote the maximum and minimum singular values. [20%]

(iii) Explain the significance of (1), (2) for control design. [15%]

(b) Outlet pressure in a compressor is controlled by manipulating the guide vanes to the compressor and using a by-pass valve. In addition to the pressure, a flow rate is measured. The plant dynamics are found to be approximated by the following transfer function:

$$G(s) = \frac{1}{(s+1)(0.2s+1)} \begin{pmatrix} e^{-0.2s} & 3 \\ 2.5e^{-0.5s} & -2.5e^{-0.3s} \end{pmatrix}.$$

An initial choice of compensator is $K(s) = K_1(s)$ where

$$K_1(s) = \begin{pmatrix} 0.25 & 0.3 \\ 0.25e^{-0.2s} & -0.1e^{-0.2s} \end{pmatrix}.$$

(i) Verify that the resulting $L(s)$ is diagonal. [15%]

(ii) The controller $K(s) = K_1(s)\mathrm{diag}(\frac{0.2s+1}{2s}, \frac{0.2s+1}{2s})$ is found to give an internally stable feedback system. [You need not verify this.] Find the frequency range for which (2) guarantees that $\overline{\sigma}(S(j\omega)) \leq 0.5$. [25%]

(iii) Without detailed calculation comment on whether there is a frequency range where $\overline{\sigma}(T(j\omega))$ is small. [10%]

2 Consider an internally stable linear multivariable feedback system as depicted in Fig. 1 in which the plant and controller have transfer functions $G(s)$ and $K(s)$ respectively.

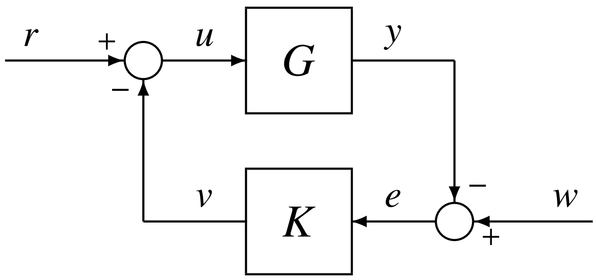

Fig. 1

(a) Show that the transfer function $H(s)$ from $\begin{pmatrix} r \\ w \end{pmatrix}$ to $\begin{pmatrix} u \\ y \end{pmatrix}$ is given by: [15%]

$$H = \begin{pmatrix} I \\ G \end{pmatrix} (I - KG)^{-1} \begin{pmatrix} I & -K \end{pmatrix}.$$

(b) Explain the importance of the quantity $b_{G,K} = \|H\|_\infty^{-1}$ in assessing the performance and robustness of feedback systems. [20%]

(c) Suppose that $u_0(t)$, $y_0(t)$ are square integrable signals such that $\hat{y}_0 = G(j\omega)\hat{u}_0$ where $\hat{}$ denotes the Fourier transform. Show that

$$H(j\omega)\begin{pmatrix} \hat{u}_0 \\ \hat{y}_0 \end{pmatrix} = \begin{pmatrix} \hat{u}_0 \\ \hat{y}_0 \end{pmatrix}. \quad (3)$$

Suppose that $r = u_0$, $w = y_0$ in Fig. 1. Write down u, y, e and v. [20%]

(d) Use (3) or otherwise to show that $b_{G,K} \leq 1$ for any G and K. [15%]

(e) Let

$$G(s) = \frac{1}{s - 1}$$

and $K(s) = -2$. Calculate $b_{G,K}$. [Hint: the non-zero eigenvalues of $ABCC^*B^*A^*$ are equal to the non-zero eigenvalues of $BCC^*B^*A^*A$ for any matrices A, B, C of compatible dimension.] [30%]

3 (a) Discuss the advantages and disadvantages of the following methods for stability analysis

(i) Lyapunov's indirect method, [10%]

(ii) Lyapunov's direct method. [10%]

(b) Consider the system

$$\dot{x}_1 = -2(x_1 + x_2)h(x_2)$$
$$\dot{x}_2 = 2x_1$$

where the function h is Lipschitz continuous and satisfies $h(z) > 1$ for all real numbers z.

(i) Use Lyapunov's indirect method to show that the origin is an asymptotically stable equilibrium point. [15%]

(ii) Consider the function

$$V(x_1, x_2) = \int_0^{x_2} z h(z) dz + x_1 x_2 + x_1^2 \quad .$$

Show that $V(x_1, x_2) > 0$ for $[x_1 \ \ x_2] \neq [0 \ \ 0]$. [15%]

(iii) Using the function $V(x_1, x_2)$ in (b)(ii), show using Lyapunov's direct method that the origin is asymptotically stable. [35%]

(iv) Is the origin also globally asymptotically stable? Justify your answer. [15%]

4 (a) Discuss the advantages and disadvantages of the circle criterion. [20%]

(b) Consider the system

$$\dot{x}_1 = x_2 - x_1 - 2g(x_1 + 2x_2)$$
$$\dot{x}_2 = -x_2 - g(x_1 + 2x_2)$$

where the function g is Lipschitz continuous and satisfies $g(0) = 0$, $zg(z) > 0$ for all real numbers $z \neq 0$.

(i) The system can be represented as the negative feedback interconnection of a linear system with transfer function $G(s)$ and the nonlinearity g. Find $G(s)$. [25%]

(ii) Use the system representation in (b)(i) to investigate whether the origin is globally asymptotically stable. [25%]

(c) Consider now the system

$$\dot{x}_1 = -x_1 - \lambda h(x_2)$$
$$\dot{x}_2 = x_1 - x_2$$

where the function h is Lipschitz continuous and is given by $h(z) = z\psi(z)$ where $\psi(z)$ satisfies $0 \leq \psi(z) \leq 1$, and $\lambda \geq 0$ is a constant. Find the range of values of λ for which global asymptotic stability of the origin can be deduced using the circle criterion. [30%]

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Answers

1. (b)(ii) Below 0.164 rad/sec.

2. (c) $u = u_0$, $y = y_0$, $e = 0$, $v = 0$.
   (e) $b_{G,K} = 1/\sqrt{10}$.

4. (b)(i) $G(s) = \dfrac{4s+5}{(s+1)^2}$.
   (c) $0 \leq \lambda < 8$.