Version JL/4

EGT3
ENGINEERING TRIPOS PART IIB

Monday 18 April 2016 9.30 to 11

Module 4F8

IMAGE PROCESSING AND IMAGE CODING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Consider a continuous image $g(u_1,u_2)$, sampled on the grid shown in Fig. 1, to produce a sampled image $g_s(u_1,u_2)$.

(i) Write down an expression for the sampling grid in Fig. 1 as the sum of two rectangular sampling grids, s_1 and s_2, denoted by black and white circles (each with a horizontal spacing of Δ and a vertical spacing of $\Delta/2$), and give the Fourier series expressions for s_1 and s_2. [30%]

(ii) Hence, if $G(\omega_1,\omega_2)$ is the spectrum of $g(u_1,u_2)$, show that the spectrum of the sampled image, $G_s(\omega_1,\omega_2)$, takes the form

$$G_s(\omega_1,\omega_2) = f(\Delta) \sum_{p_1=-\infty}^{\infty} \sum_{p_2=-\infty}^{\infty} G(\omega_1 - \beta_1 p_1, \omega_2 - \beta_2 p_2)[1+W]$$

where $f(\Delta)$, β_1 and β_2 are real functions of Δ, and W is a complex valued function of p_1 and p_2. Find f, β_1, β_2 and W. [20%]

(b) The design of 2D filters is an important part of image processing.

(i) Discuss the importance of phase in images and explain what is meant by a *zero-phase* 2D filter. Why is zero-phase a desirable property for 2D image processing filters? [15%]

(ii) A zero-phase ideal frequency response $H(\omega_1,\omega_2)$ is shown in Fig. 2. H is 1 in the shaded region and zero outside this region. The original image is sampled with spacings Δ_1 and Δ_2 in the u_1 and u_2 directions respectively.
By using standard results for lowpass/bandpass filters or otherwise, find the ideal impulse response $h(u_1,u_2)$ corresponding to $H(\omega_1,\omega_2)$. [35%]

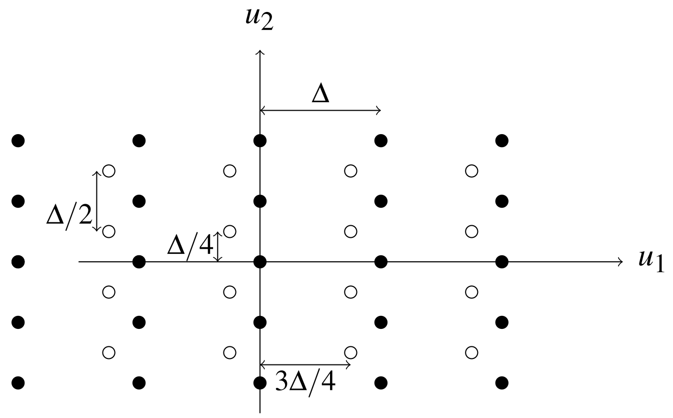
u_2
Δ
$\Delta/2$
$\Delta/4$
u_1
$3\Delta/4$

Fig. 1

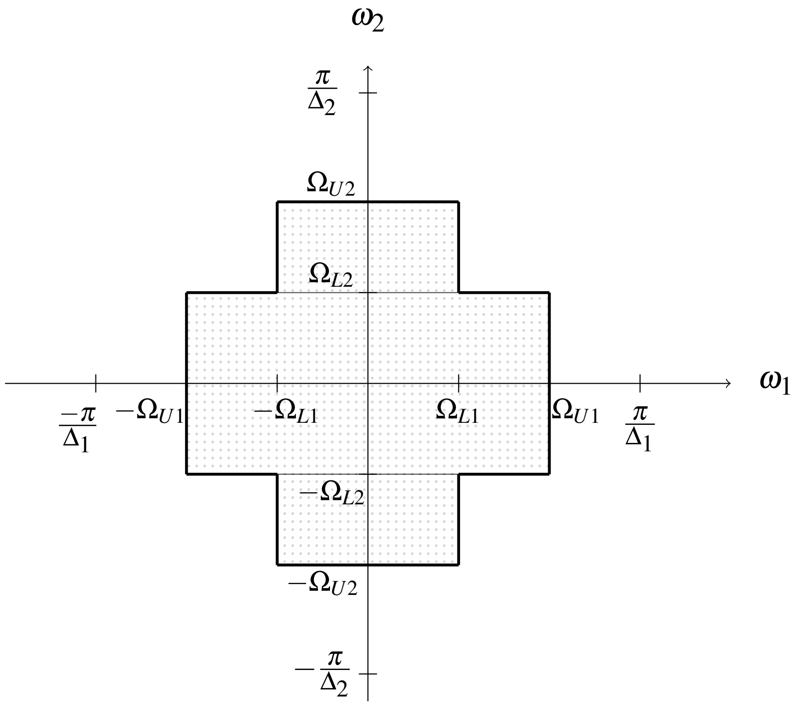
ω_2
$\frac{\pi}{\Delta_2}$
Ω_{U2}
Ω_{L2}
ω_1
$\frac{-\pi}{\Delta_1}$
$-\Omega_{U1}$
$-\Omega_{L1}$
Ω_{L1}
Ω_{U1}
$\frac{\pi}{\Delta_1}$
$-\Omega_{L2}$
$-\Omega_{U2}$
$-\frac{\pi}{\Delta_2}$

Fig. 2

2 (a) Assume that an observed image, $y(u_1, u_2)$, can be modelled as a linear distortion of the true image, $x(u_1, u_2)$, plus additive noise, $d(u_1, u_2)$.

(i) Write down an expression relating the true image to the observed image in the continuous case and then give the discrete form of this expression. Define all quantities used. [5%]

(ii) If the distorting function is known, describe how the method of *inverse filtering* can be used to estimate the true image. [10%]

(iii) Describe how we can modify the inverse filter in order to improve performance, indicating reasons why the inverse filter exhibits poor performance on real images. [5%]

(iv) In vector form we can write our true and observed images as $\mathbf{x}$ and $\mathbf{y}$ respectively. Assuming Gaussian noise, $\mathbf{d}$, and linear distortion given by the matrix L, explain why we are able to write the probability of $\mathbf{x}$ given $\mathbf{y}$ as

$$P(\mathbf{x}|\mathbf{y}) \propto \mathrm{e}^{-\frac{1}{2}[(\mathbf{y}-L\mathbf{x})^T N^{-1}(\mathbf{y}-L\mathbf{x})+\mathbf{x}^T C^{-1}\mathbf{x}]}$$

The explanation should note any assumptions made and should describe the nature of the matrices N and C. [30%]

(b) *Histogram Equalisation* is a process whereby an image displaying a poor use of grey levels is visually improved.

(i) Out of a possible range of greyscale values from 1 to 9, a 6×6 image has the form given in Fig. 3.

6	6	7	8	9	9
6	6	7	8	9	9
7	7	7	8	8	8
8	8	8	7	7	7
9	9	8	7	6	6
9	9	8	7	6	6

Fig. 3

Sketch the histogram of this image and comment on the distribution of grey levels. [10%]

(ii) Perform histogram equalisation on this image by finding the set of transformed values $\{y_k\},\ k = 1,..,9$, onto which the original greylevels are mapped. Sketch the new equalised image and its histogram, commenting on how well the process has worked. [30%]

(iii) Given the nature of this image, comment on what visual effect the above histogram equalisation produces. [10%]

3 In image compression, the discrete cosine transform (DCT) is often employed. The first row and remaining rows of a DCT matrix T for one-dimensional (1D) n-element vectors, where n is even, are given by:

$$t_{1i} = \sqrt{\frac{1}{n}} \qquad \text{for } i = 1 \ldots n$$

$$t_{ki} = \sqrt{\frac{2}{n}} \cos\left(\frac{\pi(2i-1)(k-1)}{2n}\right) \qquad \text{for } i = 1 \ldots n,\ k = 2 \ldots n$$

(a) By considering dot-products of its rows (or otherwise), show that T is an orthonormal matrix for any even value of n, and hence show that $T\,T^{\mathrm{T}} = I_n = T^{\mathrm{T}}\,T$, where I_n is an identity matrix. [20%]

(b) Show that if $Y = T\ X\ T^{\mathrm{T}}$ for $n \times n$ matrices of image pixels X and transform coefficients Y, then the energy of any X is preserved in Y. [15%]

(c) When $n = 4$ and a 2D DCT is used as the transform, it is found that for typical large real-world images, the energy of the input pixels in all of the non-overlapping $n \times n$ image blocks X becomes distributed in proportion to $(i+j-1)^{-2}$ in the 16 subimages formed by regrouping the elements of all the coefficient matrices Y, where $i = 1 \ldots 4$ and $j = 1 \ldots 4$ are the horizontal and vertical indices of the subimages. If the input image energy is σ^2 per pixel on average, determine the mean energies per DCT coefficient in each of the 16 subimages as a function of $(i+j)$. [30%]

(d) When these 16 subimages are quantised using a step size Q, the entropy in bits per coefficient is given approximately by:

$$H_{ij} = 0.5\ \log_2\left(1 + \frac{20\,E_{ij}}{Q^2}\right)$$

where E_{ij} is the mean energy per coefficient for subimage (i,j). Estimate the number of bits needed to encode an image of size 3072×2048 pixels, if the quantiser is set such that $Q = 0.5\sigma$. [20%]

(e) In the JPEG XR image compression standard, what additional process follows the 4×4 DCT (as described above) in order to further improve the energy compression stage of the transform? Explain why this tends to give better compression performance than the 8×8 DCT that is used in the baseline JPEG standard. [15%]

4 (a) Sketch the block diagram of a 2-band analysis filter bank, which is typically used as the basis for a wavelet transform, and also show the corresponding reconstruction filter bank. Explain what is meant by the *perfect reconstruction condition* for these filter banks. [20%]

(b) Show how the above analysis filter bank can be extended to produce a 2-level wavelet transform for one-dimensional (1D) signals and then describe, with the aid of a further diagram, how this 1D transform may be extended to a 2D wavelet transform for use with images. [20%]

(c) Why is it often advantageous to perform filtering on images by employing 1D filters separably on the rows and columns of an input image, rather than by convolving the image directly with a 2D filter response (or point spread function). Explain how this concept allows us to design efficient 2D wavelet transforms. [20%]

(d) $H_0(z)$ and $H_1(z)$ are the lowpass and highpass filters of a 2-band analysis filter bank, and $G_0(z)$ and $G_1(z)$ are the equivalent reconstruction filters. In order to satisfy the anti-aliasing condition, the highpass filters are usually derived from the lowpass ones using

$$H_1(z) = z^{-1} G_0(-z) \quad \text{and} \quad G_1(z) = z\, H_0(-z)$$

The perfect reconstruction (P-R) condition then requires that

$$G_0(z)\, H_0(z) + G_1(z)\, H_1(z) = 2$$

Filters are often designed employing a substitution $Z = \frac{1}{2}(z + z^{-1})$ so that G_0 and H_0 are expressed as functions of Z. Why is this useful?

If $H_0 = (1+Z)$ and $G_0 = (1+Z)(1+aZ)$, determine the value of a which produces P-R. [20%]

(e) Show how increased wavelet smoothness (more zeros at $z = -1$ in H_0 and G_0) may be achieved by adding two additional terms to the expression for Z given in part (d), and give the form of the expression to achieve this. [20%]

END OF PAPER