EGT3
ENGINEERING TRIPOS PART IIB

Monday 24 April 2017 2 to 3.30

Module 4F8

IMAGE PROCESSING AND IMAGE CODING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) The Fourier transform of a 2D image gives a 2D complex function which can be used to understand the frequency structure of the image.

(i) For the image defined by $g(u_1,u_2) = \sin(\Omega_1 u_1)\sin(\Omega_2 u_2)$, with $\Omega_1 = \pi/8$ rad s^{-1} and $\Omega_2 = \pi/16$ rad s^{-1}, sketch the Fourier transform, $G(\omega_1,\omega_2)$, of the image. [10%]

(ii) Now suppose our image takes the form $g(u_1,u_2) = \sin[\phi_1(u_1)]\sin[\phi_2(u_2)]$, where $\phi_1(u_1) = \Omega_1 u_1 + \frac{\pi}{16}u_1^2$ and $\phi_2(u_2) = \Omega_2 u_2 + \frac{\pi}{32}u_2^2$, with Ω_1 and Ω_2 as given in part (a)(i). Sketch the spectrum of this image and note how it varies from that in part (a)(i). [15%]

(iii) Discuss the relative importance of amplitude and phase in the Fourier transforms of 2D images. [10%]

(b) A continuous image $g(u_1,u_2)$ is sampled on a rectangular sampling grid with spacings Δ_1 and Δ_2 in the u_1 and u_2 directions.

(i) If this sampled image is $g_s(u_1,u_2)$, write down the 2D spectrum (Fourier transform), $G_s(\omega_1,\omega_2)$, of g_s in terms of the Fourier transform, $G(\omega_1,\omega_2)$, of the original image. [10%]

(ii) Using the result in part (b)(i), explain the phenomenon of *aliasing* in images. [10%]

(iii) A continuous image is described by the function

$$g(u_1,u_2) = \cos(\alpha u_1 + \beta u_2)$$

Form the Fourier transform, $G(\omega_1,\omega_2)$, of $g(u_1,u_2)$ and verify that g is bandlimited. [20%]

(iv) Find the Nyquist sampling frequencies for the image in part (b)(iii). [10%]

(v) What are the values of α and β in part (b)(iii) if sampling spacings of $\Delta_1 = 0.4\pi$ and $\Delta_2 = 0.2\pi$ are the Nyquist sampling intervals? [15%]

2 (a) Filtering 2D images involves convolving the image with an *impulse response* (IR) or *point spread function* (PSF). In the majority of cases we assume that the filter is spatially invariant.

(i) Explain what is meant by a *zero-phase* filter and say why this is a desirable property for processing images. [10%]

(ii) We wish to design a filter with a desired zero-phase 2D frequency response (for example, a rectangular low-pass filter). Explain why *windowing* is necessary and describe the effect of windowing on the desired frequency response. [10%]

(iii) Describe two methods of forming 2D window functions from 1D window functions. [10%]

(b) An observed image $\mathbf{y}$ can often be modelled as the original image $\mathbf{x}$ convolved with a spatially invariant filter, $\mathbf{h}$, plus noise, $\mathbf{n}$:

$$\mathbf{y} = \mathbf{h} * \mathbf{x} + \mathbf{n}$$

[$*$ denotes convolution]

(i) Explain how, neglecting noise, one can apply *inverse filtering* to extract the original image from $\mathbf{y}$, given knowledge of $\mathbf{h}$. Comment on the performance of such inverse filtering methods in the presence of noise. [10%]

(ii) If a camera images a moving object (e.g. a chessboard pattern), by opening its shutter for T seconds, explain why the resulting image $y(u_1, u_2)$ may be represented by

$$y(u_1, u_2) = \int_0^T x(u_1 - vt, u_2)dt$$

where we assume that $x(u_1, u_2)$ models the object and the speed of the object is v in the u_1 direction and zero in the u_2 direction. [15%]

(iii) Show that the motion blur in part (b)(ii) can be modelled by $\mathbf{y} = \mathbf{h} * \mathbf{x} + \mathbf{n}$, and find the filter $\mathbf{h}$. [20%]

(iv) Neglecting noise, construct a simple deblurring filter for this motion blur and discuss the performance of this filter. [15%]

(v) Discuss (qualitatively) why the Wiener filter should produce better results than inverse filtering, and how one can do even better by non-linear deconvolution techniques. [10%]

3 (a) A coding system for achieving image compression comprises an encoder and a decoder, each of which may be split into three main processing blocks. Draw a block diagram of the system and briefly explain the function of each block. [20%]

(b) In a particular coding system, a 4-point discrete cosine transform (DCT) is used to analyse the input image and reconstruct the output image. Explain why it is desirable for the one-dimensional transform matrix T to be orthonormal (unitary). Show how T may be used to calculate the two-dimensional (2D) transform Y of a 4×4 block of image pixels X, and also how X may be reconstructed from any given 4×4 block of coefficients Y. [20%]

(c) The matrix T may be expressed as

$$T = \begin{bmatrix} a & a & a & a \\ b & c & -c & -b \\ a & -a & -a & a \\ c & -b & b & -c \end{bmatrix} \quad \text{where} \quad \begin{cases} a = 1/2 \\ b = \cos(\pi/8)/\sqrt{2} \\ c = \cos(3\pi/8)/\sqrt{2} \end{cases}$$

Show that T is orthonormal and obtain expressions, in terms of a, b and c, for the basis functions of the 2D DCT corresponding to coefficients $y_{1,1}$, $y_{1,2}$, $y_{2,1}$ and $y_{2,2}$ in the upper left quarter of Y. (Hint: you may obtain the basis functions by calculating the pixel matrices X, which correspond to coefficient matrices Y in which just a single coefficient is unity and the rest are zero.) [20%]

(d) Without further calculation, describe the form of the remaining 12 of the 16 basis functions of the 4×4-point 2D DCT. Suggest reasons why, for images of the real world, the entropies of the 16 resulting DCT subbands, indexed by (i, j), tend to decrease as $(i + j)$ increases from 2 to 8. [20%]

(e) Further reductions in the number of bits of the encoded images may be achieved by choosing the quantiser step-sizes independently for each DCT subband. Explain what property of the human visual system allows this to give improved tradeoffs of perceived image quality versus coded bit-rate (compared with a uniform step-size for all subbands), and how this relates to the 16 DCT basis functions. [20%]

4 (a) A two-band filter bank for analysing one-dimensional (1D) signals comprises filters $H_0(z)$ and $H_1(z)$. The corresponding filter bank for reconstructing signals comprises filters $G_0(z)$ and $G_1(z)$. The analysis filter outputs are down-sampled by 2 by discarding alternate samples, and the inputs to the reconstruction filters are up-sampled by 2 by insertion of zeros in the alternate sample positions. Sketch a block diagram of this filter-bank system and explain why the down and up samplers are used for applications involving signal compression. [20%]

(b) Show that a down-sampler by 2, followed by an up-sampler by 2, converts a signal with z-transform $Y(z)$ into a signal with z-transform $[Y(z)+Y(-z)]/2$. [15%]

(c) Hence, for the system described in part (a), obtain an expression for the output $\hat{X}(z)$ of the reconstruction filter bank in terms of the four filters, H_0, H_1, G_0 and G_1, and the input X to the analysis filter bank. From this, obtain the two conditions on the four filters to achieve perfect reconstruction of X at the output $\hat{X}$. [20%]

(d) Show how the block diagram of the two-band analysis filter bank may be extended to achieve a 1D wavelet transform over 2 scales (or levels), and then sketch the block diagram for a standard two-dimensional (2D) wavelet transform over the same number of scales. (Assume that H_0 is a lowpass filter and that H_1 is highpass; and similarly for G_0 and G_1.) Describe briefly what simple features of an input image are picked out by each 2D wavelet subband. [25%]

(e) Explain how the perfect reconstruction property for a simple filter-bank system, as derived in part (c), may be applied to the 2D wavelet transform system of part (d) to ensure perfect image reconstruction in the absence of quantising effects; and also briefly discuss desirable properties for G_0 and H_0 if quantising artefacts are to have low human visibility. [20%]

END OF PAPER

THIS PAGE IS BLANK