EGT3
ENGINEERING TRIPOS PART IIB

Thursday 21 April 2016 9.30 to 11.00

Module 4G6

CELLULAR AND MOLECULAR BIOMECHANICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3C7 datasheet (2 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 The biological network shown in Fig. 1 can be treated as a periodic assembly of elastic-plastic struts with cell wall elastic modulus E_s and yield strength σ_{YS}. The struts are of uniform thickness t.

(a) Obtain an expression for the relative density of the network. [20%]

(b) Calculate the modulus and yield strength for uniaxial tension in the x_2 direction. [20%]

(c) The effective strength of the network for uniaxial loading in the x_1 direction is dictated by plastic bending of hinges that form at the ends of each inclined strut.

(i) Relate the rotation rate of these hinges to the applied macroscopic strain rate $\dot{\varepsilon}_{11}$ in the x_1 direction. [30%]

(ii) Determine the yield strength of the network for uniaxial loading in the x_1 direction. [30%]

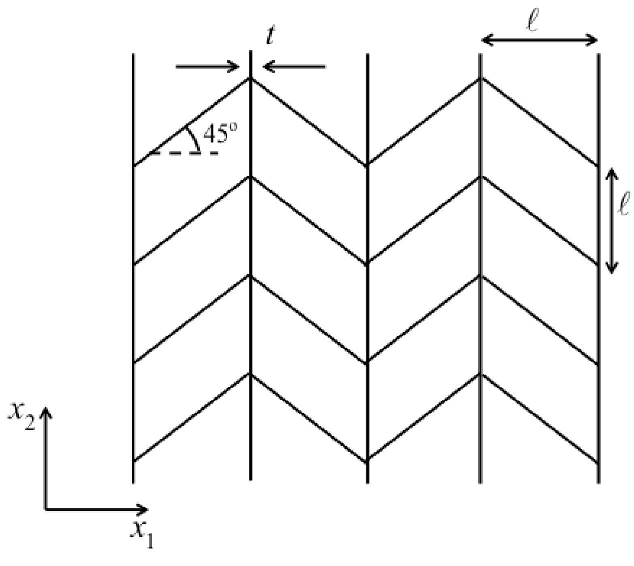

Fig. 1

2 (a) Physiologists have long known that muscle speed v decreases with increasing load T according to the Hill equation

$$(T+a)v = b(T_o - T)$$

where a and b are constants and T_o the isometric tetanic tension. Determine the speed at which the muscle power is maximised. [30%]

(b) In the Huxley sliding filament model for a muscle, the fraction $n(x)$ of attached crossbridges is given by

$$n(x) = \begin{cases} n_o \exp\left(\dfrac{kx}{v}\right) & x < 0 \\ n_o & 0 \le x \le h \\ 0 & x > h \end{cases}$$

where n_o and k are constants, x is the position of an actin binding site from the equilibrium position of a myosin head and $v \equiv -\mathrm{d}x/\mathrm{d}t$ is the shortening velocity of the muscle. Consider a muscle that has a cross-sectional area A, sarcomere length s, and m crossbridges per unit volume. Assume that a linear spring with stiffness λ connects the myosin head to the thick filament.

(i) Determine the tension-velocity relation for this muscle. You may assume that the M and A sites are spaced a distance $l >> h$ apart. [50%]

Hint: $\int x\mathrm{e}^{qx}\mathrm{d}x = \dfrac{1}{q^2}\left[qx\mathrm{e}^{qx} - \mathrm{e}^{qx}\right]$

(ii) Briefly discuss the quality of the agreement of this model with the Hill equation. [20%]

3 Explain the following:

(a) The cell membrane of red blood cells has a very low elastic modulus, but a large lock-up strain and a high ultimate strength. [25%]

(b) Wood is strongly anisotropic, with a compressive strength along the grain an order of magnitude higher than that across the grain. [25%]

(c) Proteins are transported within cells at much faster rates than diffusion can provide. [25%]

(d) Plant and animal cells have different strategies for harvesting energy. [25%]

4 (a) Explain the physical basis of persistence length in a biological fibre and give examples where (i) the structural length is much greater than and (ii) much less than the persistence length. [30%]

(b) Why is the cell wall dominant in dictating the mechanical properties of plant cells while the cytoskeleton dictates the response of animal cells? [20%]

(c) Outline the structure of a sarcomere and explain the role of thick filaments, thin filaments and the Z-discs. [30%]

(d) Discuss the mechanism of glucose transport across the cell membrane and how insulin affects this transport rate. [20%]

END OF PAPER

Numerical answers

1. (a) $\bar{\rho} = (1+\sqrt{2})(t/\ell)$

   (b) $E_2 = \frac{\bar{\rho}}{(1+\sqrt{2})} E_s$; $\sigma_{2Y} = \frac{\bar{\rho}}{(1+\sqrt{2})} \sigma_{YS}$

   (c)(ii) $\Sigma_{11} = \frac{\bar{\rho}^2}{2(1+\sqrt{2})^2} \sigma_{YS}$

2. (a)

$$v_{opt} = \frac{b\left[1 + \frac{a}{T_0} - \sqrt{\left(\frac{a}{T_0}\right)^2 - \frac{a}{T_0}}\right]}{\sqrt{\left(\frac{a}{T_0}\right)^2 + \frac{a}{T_0}}}$$

   (b)

$$T = \frac{n_0 s \lambda}{2\ell}\left[\frac{h^2}{2} - \left(\frac{v}{k}\right)^2\right]$$