Version GTP/4

EGT3 / EGT2
ENGINEERING TRIPOS PART IIB
ENGINEERING TRIPOS PART IIA

Friday 22 April 2016   2 to 3.30

Module 4M16

NUCLEAR POWER ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4M16 data sheet (8 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Starting from the general form of the neutron diffusion equation given on page 6 of the 4M16 data sheet, derive the neutron diffusion equation for a steady-state, source-free system

$$\nabla^2\phi + \frac{(\eta - 1)\Sigma_\mathrm{a}}{D}\phi = 0$$

stating any assumptions made. [15%]

(b) In spherical geometry the solution to this equation is

$$\phi(r) = \frac{A}{r}\sin(Br)$$

where $B^2 = \dfrac{(\eta - 1)\Sigma_\mathrm{a}}{D}$ and A is a constant. Show that the equivalent solution for a *cubic* reactor in Cartesian geometry is

$$\phi(x,y,z) = C\cos\left(\frac{B}{\sqrt{3}}x\right)\cos\left(\frac{B}{\sqrt{3}}y\right)\cos\left(\frac{B}{\sqrt{3}}z\right)$$

where C is another constant. [35%]

(c) A large thermal reactor is to be constructed. From reactor physics measurements the value of the parameter B^2 of the proposed fuel-moderator-coolant (lattice) configuration is found to be $0.25\ \mathrm{m}^{-2}$. Stating any assumptions made, estimate the minimum critical volume for an *unreflected* core with this lattice. What should its shape be? [20%]

(d) The reactor is actually to be built in the form of a rectangular parallelepiped (cuboid) with a graphite reflector on all sides. Estimate the new minimum volume of core material. You can assume that the reflector saving arising from the use of the reflector is equal to the diffusion length $L = \sqrt{D/\Sigma_\mathrm{a}}$ in the reflector, for each reflector face. Relevant data for graphite can be found on page 7 of the 4M16 data sheet. [30%]

2 In a 'lumped' model of the behaviour of a source-free reactor, the equations for the neutron population n and the precursor population c can be written as

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c$$

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c$$

where all symbols have their usual meanings.

(a) What major simplifying assumptions underlie this model? [10%]

(b) Derive the relationship between the neutron and precursor populations when the reactor is in equilibrium. What is the value of the reactivity ρ in this case? [10%]

(c) A critical, source-free reactor has been operating in equilibrium for a prolonged period. It is then subject to a step increase in reactivity at time $t = 0$. You can assume without proof that, for this model, the *in-hour equation* relating the inverse periods p of the subsequent kinetic behaviour to the reactivity ρ is

$$\rho = p\left[\Lambda + \frac{\beta}{(p + \lambda)}\right]$$

(i) Sketch the relationship between the values of p that satisfy this equation and ρ, identifying any asymptotes. [15%]

(ii) Find the dominant time constant of the resulting excursion for the case where $\rho = 0.005$, $\beta = 0.0075$, $\lambda = 0.1\ \mathrm{s}^{-1}$ and $\Lambda = 10^{-4}$ s. [15%]

(d) Show that the variation of the neutron population predicted by the *prompt jump approximation* is

$$n(t) = \frac{\beta}{\beta - \rho} n_0 \exp\left(\frac{\rho\lambda}{\beta - \rho} t\right)$$

where n_0 is the steady-state neutron population prior to the increase in reactivity and ρ is the reactivity level after the increase. [40%]

(e) Find the dominant time constant predicted by the prompt jump approximation for the case in (c)(ii) and comment on the safety implications of using this estimate. [10%]

3 (a) What are the main advantages and disadvantages of on-line refuelling? [10%]

(b) The reactivity ρ of a particular design of Pressurized Water Reactor (PWR) fuel varies linearly with burn-up τ as

$$\rho = \rho_0 \left(1 - \frac{\tau}{T_1}\right)$$

where ρ_0 is the initial reactivity of the fuel and T_1 is the burn-up at which the reactivity of the fuel falls to zero. In the planned *M*-batch refuelling scheme, one *M*-th of the fuel is to be changed at the beginning of each cycle, where *M* is an integer. The same fuel design (enrichment) will be used throughout.

Assuming an initial loading of all fresh fuel of the same design, find the cycle lengths (measured in units of burn-up) for the first and second cycles of operation, and derive an equation showing the dependence of the length of the *M*-th cycle on its predecessors. State any assumptions made. [30%]

(c) Show that the ratio of the equilibrium cycle length in *M*-batch operation T_M to the cycle length in one-batch operation T_1 is given by

$$\frac{T_M}{T_1} = \frac{2}{M+1}$$ [15%]

(d) A four-batch reloading scheme is chosen for this PWR. Show that, in equilibrium operation, the fuel utilisation is only 80% of the utilisation available with continuous on-line refuelling. [15%]

(e) The whole-core reactivity of the fuel is proportional to the quantity $(e - 1.5)$ where e is the ^{235}U enrichment percentage. In equilibrium operation the enrichment of the fuel is to be 3.5%, i.e. $e = 3.5$. If, rather than having varying cycle lengths as in (b), the equilibrium batch sizes and cycle lengths are to be employed immediately for the four-batch refuelling strategy, recommend enrichments for the initial batches at start-up. [30%]

4 (a) Why is it necessary to enrich the ^{235}U content of the fuel for most designs of civil reactor? Give typical values of the enrichment required. [10%]

(b) Describe in detail the process most commonly used today for the enrichment of civil nuclear fuel, and briefly describe one historic method and one method under development. [30%]

(c) Define the term *separative work unit*. Explain how separative work is calculated. What is the main criterion used in determining the tails concentration of an enrichment process? [20%]

(d) A 1200 MW(e) reactor requires fuel enriched to 4.0%. The reactor has an overall thermal efficiency of 30% and operates with an availability of 85%. The discharge burnup of the fuel is $40\,\mathrm{GWd\,te^{-1}}$. Estimate the annual fuel requirement. Assume that the ^{235}U content of natural uranium is 0.7% and that the uranium ore concentrate (UOC) has a concentration of 95% U. Estimate how much UOC will be required if the tails concentration is 0.3% ^{235}U. Ignore losses at all stages of the fuel cycle except enrichment process tails. How much separative work will be needed annually? [40%]

END OF PAPER

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Answers

Q1 (c) A sphere of volume 1039 m^3

(d) A cube of volume 942 m^3

Q2 (b) $\frac{\beta}{\Lambda} n_0 = \lambda c_0;\ \rho = 0$

(c)(ii) 5.0595 s

(e) 5.0 s

Q3 (b) $\tau_1 = T_1;\ \tau_2 = \frac{T_1}{M};\ \tau_M = T_1 - \frac{1}{M}\sum_{i=1}^{M-1} i\tau_i$

(e) Batches with enrichments of 3.5%, 2.7%, 1.9% and 1.1%

Q4 (d) $P = 31.025 \text{ te};\ M_{\text{UOC}} = 302.09 \text{ te};\ S = 162.82 \text{ te SWU}$