EGT3 / EGT2
ENGINEERING TRIPOS PART IIB
ENGINEERING TRIPOS PART IIA

Friday 28 April 2017 2 to 3.30

Module 4M16

NUCLEAR POWER ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4M16 data sheet (8 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Radioembolization is a nuclear medicine procedure to treat liver cancer. Tiny glass or resin beads filled with the radioactive isotope yttrium-90 are placed inside the blood vessels that feed a tumour. This blocks the supply of blood to the cancer cells and delivers a high dose of radiation to the tumour while sparing normal tissue.

(a) Yttrium-90 decays by pure β^- emission into zirconium-90. ^{90}Y has an atomic mass of 89.907152 u. ^{90}Zr has an atomic mass of 89.904703 u. Show that the energy released in the decay reaction is 2.28 MeV. [5%]

(b) ^{90}Y has a half-life of 64.05 hours. Calculate its specific activity (in $\mathrm{Bq\,kg^{-1}}$). [10%]

(c) Glass beads used in radioembolization are spherical with a diameter of 25 μm. If ^{90}Y occupies 5% of the bead volume, calculate the maximum possible activity of one bead. The density of yttrium is $4.742 \times 10^3\ \mathrm{kg\,m^{-3}}$. [15%]

(d) The maximum β radiation energy from ^{90}Y is 2.28 MeV, as shown in (a). The average β radiation energy is 0.94 MeV. The average and maximum ranges of the ^{90}Y β radiation in tissue are 2.5 and 11 mm, respectively. Given that the radiation weighting factor for β radiation is 1, estimate the *absorbed dose* and the *equivalent dose* received by the tissue surrounding a single glass bead. Assume that, to account for the effect of the delay between bead manufacture and deployment, the initial activity of the bead is only 10% of that calculated in (c). State and justify any other assumptions and approximations you make. [40%]

(e) The organ weighting factor for the liver is 0.05. A typical human liver weighs 1.35 kg. If 1000 beads are delivered to the area of the cancerous tumour in a radioembolization procedure, estimate the *absorbed dose*, the *equivalent dose* and the *effective dose* to the liver from this procedure. [10%]

(f) Discuss the significance of your dose calculations in (d) and (e) and their implications for the bead delivery process. What characteristics of ^{90}Y make it well suited for internal radionuclide therapy of this sort? [20%]

2 (a) Using the information on page 6 of the 4M16 data sheet, show that the one-group, steady-state, source-free neutron diffusion equation for a homogeneous, non-multiplying medium with spherical symmetry can be written as

$$\frac{1}{r}\left(\frac{d^2(\phi r)}{dr^2}\right) - \frac{\phi}{L^2} = 0$$

defining L^2 and stating any assumptions made. [25%]

(b) A large spherical mass of this material has a central cavity of radius R_1 at the centre of which is an isotropic source of neutrons causing the flux at R_1 to be ϕ_1. Show that the neutron flux distribution throughout the material is given by

$$\phi = \frac{\phi_1 R_1}{r}\exp\left(\frac{R_1 - r}{L}\right)$$ [30%]

(c) Show how the steady-state, source-free diffusion equation should be modified if it applies to a multiplying medium. [5%]

(d) If the infinite multiplication factor k_∞ of the multiplying medium is greater than unity, derive an expression for the critical radius of a solid (i.e. with no central cavity), bare, spherical reactor made of this material. Explain carefully the boundary condition you have used at the edge of the reactor. [40%]

3 An Advanced Gas-cooled Reactor (AGR) fuel element is rated at 10 MW and is 8 m long. The equivalent bare reactor is 9 m long. The coolant enters at 335 °C and leaves at 635 °C. The effective diameter of the fuel in the channel is 0.8 m and the overall cladding-to-gas heat transfer coefficient is $5\,\mathrm{kW m^{-2} K^{-1}}$. The power distribution along the fuel element can be assumed to have the form of a 'chopped cosine'.

(a) Which cladding material is used in AGRs? Which gas is used as the coolant? [5%]

(b) Sketch the forms of the variation of the coolant, cladding and fuel temperatures along the channel. [20%]

(c) Show that the maximum non-dimensional temperature $\theta_{\max}$ given by Ginn's equation occurs at a distance along the channel

$$x = \frac{2L'}{\pi} \tan^{-1}\left(\frac{1}{Q}\right)$$

where all symbols have their usual meanings. [10%]

(d) Determine the location and the magnitude of the maximum cladding surface temperature along the channel. You can assume there is no scale on the cladding. [45%]

(e) Another fuel element in the same reactor, further from the centre of the core, is rated at 5 MW. It is proposed to 'gag' the coolant in the channel, i.e. to reduce the coolant mass flow rate through the channel. Assume that the coolant entry temperature, the coolant specific heat capacity and the overall cladding-to-gas heat transfer coefficient are unchanged by any gagging.

(i) Compared to the 10 MW channel, what is the fractional reduction in coolant mass flow rate needed to achieve the same coolant exit temperature? [5%]

(ii) If this condition is achieved, will the maximum cladding surface temperature along the 5 MW channel be greater or less than that along the 10 MW channel? Explain your reasoning. [15%]

4 The equations governing the behaviour of xenon-135 in a 'lumped' reactor model can be written as

$$\frac{dI}{dt} = \gamma_i \Sigma_f \phi - \lambda_i I$$

$$\frac{dX}{dt} = \gamma_x \Sigma_f \phi + \lambda_i I - \lambda_x X - \sigma X \phi$$

where all symbols have their usual meanings.

(a) Explain the physical meaning of each of the four terms on the right-hand side of the second equation. [10%]

(b) Show that the steady-state poisoning effect of the xenon-135 is given by

$$\rho_{\text{Xe0}} = -\frac{\sigma(\gamma_x + \gamma_i)\phi}{\nu(\lambda_x + \sigma\phi)}$$

where ν is the average number of neutrons released in fission. [20%]

(c) If the maximum excess reactivity available from the control rods is 0.02, find the highest neutron flux level ϕ_{max} that can be sustained in the reactor. Take $\gamma_i = 0.061$, $\gamma_x = 0.003$, $\lambda_i = 2.874 \times 10^{-5}\ \text{s}^{-1}$, $\lambda_x = 2.093 \times 10^{-5}\ \text{s}^{-1}$, $\sigma = 2.75\ \text{Mb}$ and $\nu = 2.43$. [10%]

(d) The reactor is shut down after prolonged operation at flux level ϕ_{max}. Show that the subsequent time variation of the xenon-135 population is given by

$$X = \frac{(\gamma_x + \gamma_i)\Sigma_f \phi_{\text{max}}}{\lambda_x + \sigma\phi_{\text{max}}} \exp(-\lambda_x t) + \frac{\gamma_i \Sigma_f \phi_{\text{max}}}{\lambda_x - \lambda_i}\left[\exp(-\lambda_i t) - \exp(-\lambda_x t)\right]$$

where time t is measured from the point of shutdown. [45%]

(e) Show that the reactor cannot be restarted again until a time t given by the solution of the equation

$$\frac{\gamma_x + \gamma_i}{\lambda_x + \sigma\phi_{\text{max}}}\left[1 - \exp(-\lambda_x t)\right] = \frac{\gamma_i}{\lambda_x - \lambda_i}\left[\exp(-\lambda_i t) - \exp(-\lambda_x t)\right]$$

and comment on the practical implications of this. [15%]

END OF PAPER

4M16 Nuclear Power Engineering 2017

Answers

Q1 (b) 2.014×10^{19} Bq kg^{-1}

(c) 39.07×10^{6} Bq

(d) 35.2 Gy; 35.2 Sv

(e) 145.2 Gy; 145.2 Sv; 7.26 Sv

Q2 (c) $D\nabla^2\phi + (\eta - 1)\Sigma_\mathrm{a}\phi = 0$

(d) $\dfrac{\pi}{B_m} - \delta$ where δ is the extrapolation distance and $B_m^2 = \dfrac{(\eta - 1)\Sigma_\mathrm{a}}{D}$

Q3 (a) Stainless steel cladding; carbon dioxide coolant

(d) 692.6°C at 2.36 m past the channel centre

(e)(i) 50% reduction

(ii) Lower

Q4 (c) 2.402×10^{17} m^{-2}s^{-1}