Version DJC/3

EGT1
ENGINEERING TRIPOS PART IB

Monday 30 May 2016 9 to 11

Paper 1

MECHANICS

*Answer not more than **four** questions.*

*Answer not more than **two** questions from each section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

*Answer not more than **two** questions from this section*

1 Figure 1 shows in plan view the steering mechanism of one wheel of a vehicle. The wheel is attached to member CDE which is connected to the stationary frame AB of the vehicle by two pivoted links AC and BD. Steering is accomplished by extending or contracting the hydraulic actuator BE. Member CDE is at the given instant parallel to AB, and member BD has constant angular velocity ω in the clockwise direction.

(a) Draw a velocity diagram for the mechanism at the instant shown (suggested scale ωa = 25 mm). Find the sliding velocity of the actuator piston relative to the cylinder, and the angular velocities of CDE, AC and BE. [10]

(b) Draw an acceleration diagram for the mechanism at the instant shown (suggested scale $\omega^2 a$ = 20 mm). Find the sliding acceleration of the actuator piston relative to the cylinder. [15]

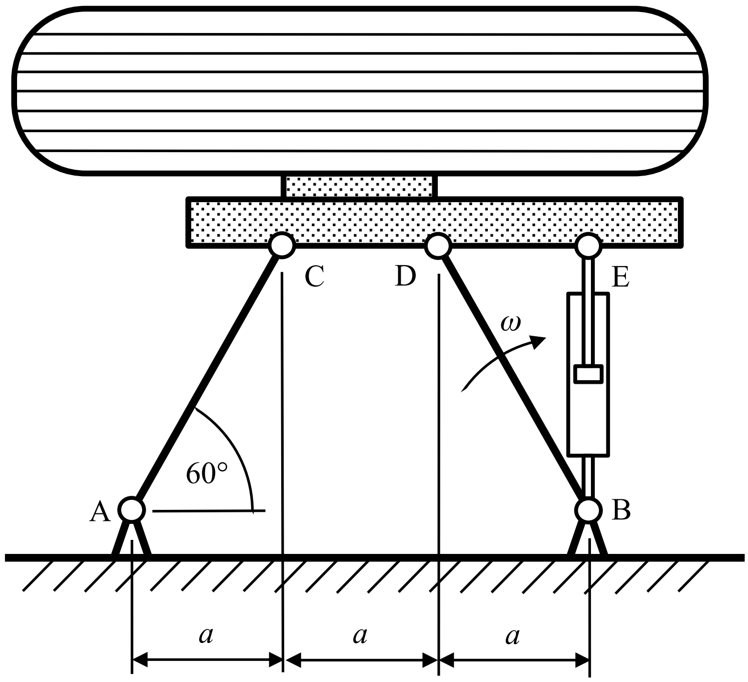

Fig. 1

2 A thin rigid uniform plank rests symmetrically on two horizontal rigid rails A and B that are at the same height, distance $2a$ apart and perpendicular to the plank. The plank has mass m and its moment of inertia about the centre of mass is mk^2 where k is the radius of gyration. One end of the plank is raised so that the centre of mass rises by a *small* height h_1, as shown in Fig. 2, and is then released. The plank rocks from one rail to the other. At any instant the plank is in contact with one or other rail. No sliding takes place at the contacts.

Note: the small height h_1 means that the usual small angle approximations can be made: $\sin\theta = \tan\theta = \theta$ and $\cos\theta = 1$.

(a) Derive an expression for the time T_1 taken for the plank to fall from height h_1 until the first impact with one of the rails. [7]

(b) Derive an expression for the ratio of angular velocities of the plank immediately before and after an impact. Hence derive an expression for the ratio of the greatest heights achieved before and after an impact. [8]

(c) Using your answers to (a) and (b), or otherwise, derive an expression for the time taken for the plank to stop rocking if $k = 2a$. [10]

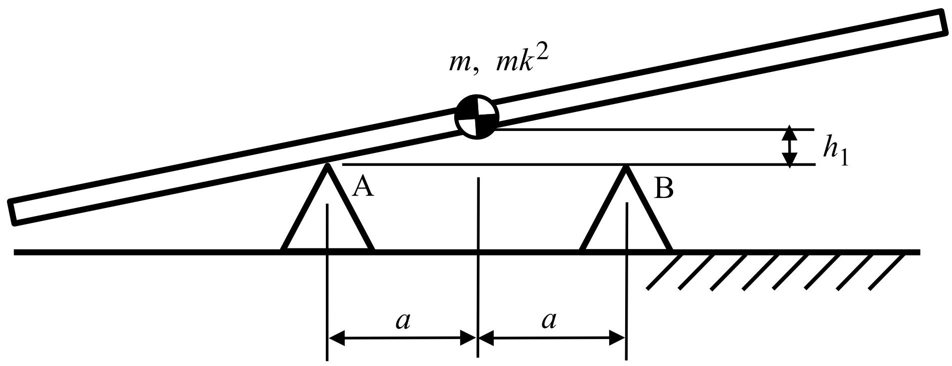

Fig. 2

3 Figure 3 shows in plan view a piston D connected to a crank AB by a connecting rod BCD with frictionless pinned joints at B and D. The piston has mass m_P and a friction force F opposes sliding motion of the piston in its cylinder. The connecting rod has mass m_R centred at C, which is halfway along BD, and has rotational inertia I about C. The crank has length a and rotates at constant angular velocity ω in the anti-clockwise direction. At the instant shown the angle DAB is 90° and the angle ABD is 60°. For the instant shown:

(a) find the velocity of the piston, the angular velocity of the connecting rod and the translational velocity of the connecting rod at C. The suggested scale for a velocity diagram is ωa = 50 mm. [6]

(b) find the acceleration of the piston, the angular acceleration of the connecting rod and the translational acceleration of the connecting rod at C. The suggested scale for an acceleration diagram is $\omega^2 a$ = 100 mm. [9]

(c) find the torque at the crank necessary to maintain crank rotation at constant angular velocity ω. [10]

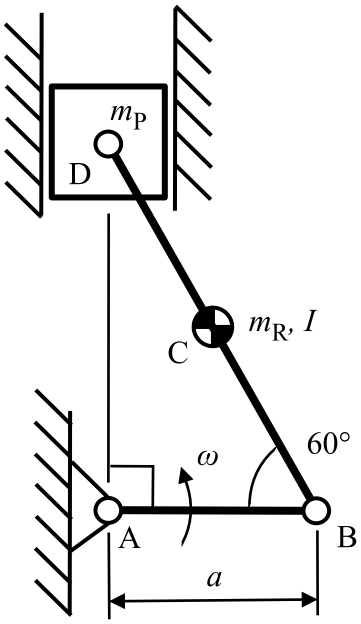

Fig. 3

SECTION B

*Answer not more than **two** questions from this section.*

4 (a) Explain the terms *static balance* and *dynamic balance*, as applied to rotating shafts. If a shaft carries three imbalanced rotors, under what conditions can the shaft be statically balanced by rotating the rotors relative to one another? [5]

(b) The shaft shown in Fig. 4 is supported by two bearings X and Y and carries three rotors A, B and C of diameter 0.2 m in the positions shown. The imbalances of rotors A, B and C are 12, 9 and 15 kg mm respectively along the lines shown on the rotors. The shaft rotates at 5000 revolutions per minute.

(i) How should rotors B and C be rotated relative to rotor A to achieve static balance? [5]

(ii) What will be the magnitude of the reaction forces at the bearings when the rotors are statically balanced as in (i)? [10]

(iii) How much material should now be removed from the outer edges of rotors A and C, and at what angular positions, to achieve dynamic balance? [5]

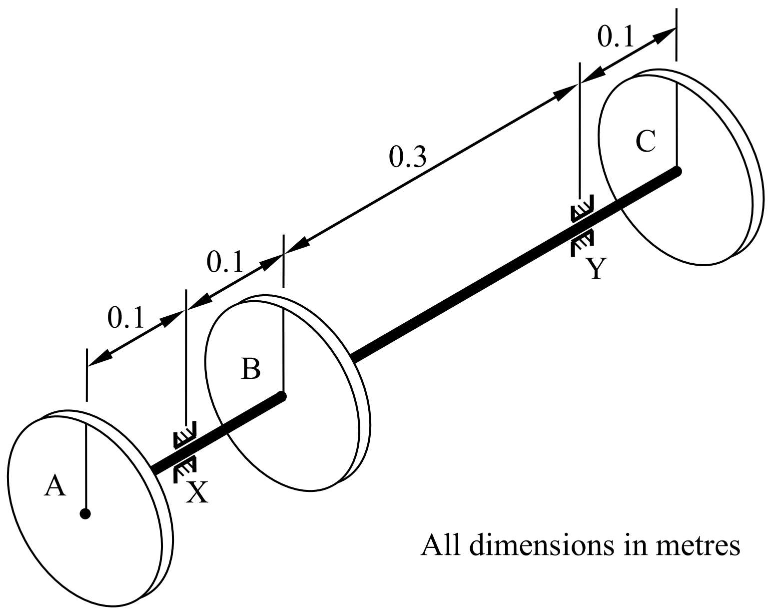

Fig. 4

5 A simple two-speed gearbox is shown in Fig. 5. Shafts 1, 2 and 3 are mounted in rigid bearings and are axially constrained. Shaft 4 is mounted inside shaft 3, such that it is able to slide relative to shaft 3, and also to rotate at a different angular velocity. When shaft 4 slides to the left, clutch plates X and Y engage, so that shafts 1 and 4 rotate at the same angular velocity; when it slides to the right, clutch plates Y and Z engage, so that shafts 3 and 4 rotate at the same angular velocity. Shafts 1 and 4 carry rotors with polar inertias of J and $2J$ respectively, as shown; the shafts themselves and the gears all have negligible inertia. Gear wheels A, B, C, and D have 12, 36, 18 and 30 teeth respectively.

The system is set up with clutch plates X and Y engaged, and shaft 1 is given angular velocity ω. With the system rotating freely, shaft 4 is then moved to the right, so that clutch plate Y disengages from X and engages with Z.

(a) Show that the speed of shaft 1 is 5 times the speed of shaft 3. [4]

(b) Explain why the total angular momentum of the system about the axis of the rotors is not conserved when slipping at the clutch has ceased. [4]

(c) Explain why the impulse between gears C and D must be twice as large as that between gears A and B. [4]

(d) Find the angular velocity of shaft 1 when slipping at the clutch has ceased. [10]

(e) Find the fraction of initial energy lost when slipping at the clutch has ceased. [3]

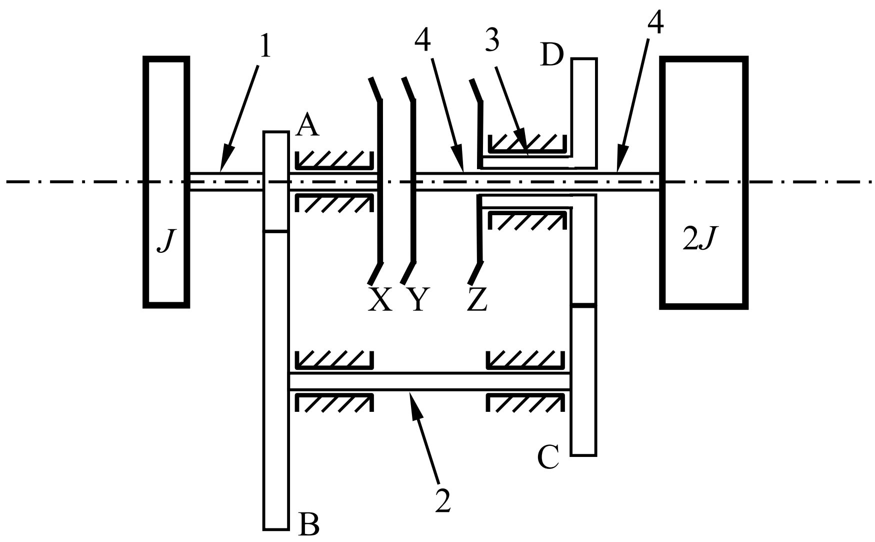

Fig. 5

6 A uniform bar of length l and mass m is freely pivoted to a fixed bearing, as shown in elevation in Fig. 6. The free end of the bar is held vertically above the pivot and is then released, allowing the bar to rotate under the influence of gravity. For the instant when the bar is horizontal (that is, when $\theta = 90°$), find the:

(a) angular acceleration of the bar; [5]

(b) angular velocity of the bar; [5]

(c) horizontal and vertical components of the reaction at the pivot; [5]

(d) position and value of the maximum bending moment in the bar. [10]

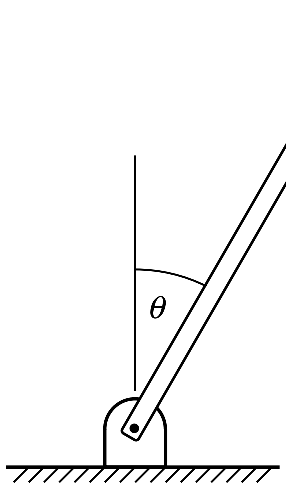

Fig. 6

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Answers

1 (a) $3\omega a$ extending; 2ω anticlockwise; ω clockwise; ω clockwise

(b) $2\sqrt{3}\,\omega^2 a\uparrow$

2 (a) $\sqrt{\frac{2h_1(k^2+a^2)}{ga^2}}$

(b) $\frac{\omega_{\text{after}}}{\omega_{\text{before}}}=\frac{k^2-a^2}{k^2+a^2}$ $\frac{h_{\text{after}}}{h_{\text{before}}}=\left(\frac{k^2-a^2}{k^2+a^2}\right)^2$

(c) $4\sqrt{\frac{10h_1}{g}}$

3 (a) $\omega a\uparrow$
zero
$\omega a\uparrow$

(b) $\frac{\omega^2 a}{\sqrt{3}}\uparrow$
$\frac{\omega^2}{\sqrt{3}}$ clockwise
$\frac{\omega^2 a}{2\sqrt{3}}\uparrow$ $\frac{\omega^2 a}{2}\leftarrow$

(c) $m_{\text{P}}\frac{\omega^2 a^2}{\sqrt{3}}+m_{\text{R}}\frac{\omega^2 a^2}{2\sqrt{3}}+Fa$ anticlockwise

4 (b) (i) 90°, 216.9°

(ii) 5517 N

(iii) 134 g A: 26.6° C: 206.6°

5 (d) $\frac{35}{27}\omega$

(e) $\frac{32}{81}$

6 (a) $\frac{3g}{2l}$

(b) $\sqrt{\frac{3g}{l}}$

(c) $\frac{3}{2}mg\leftarrow$ $\frac{1}{4}mg\uparrow$

(d) $\frac{mgl}{27}$ sagging, $\frac{l}{3}$ from pivot