EGT1
ENGINEERING TRIPOS PART IB

Monday 30 May 2016   2 to 4

Paper 2

STRUCTURES

*Answer not more than **four** questions, which may be taken from either section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

1 A pin-jointed truss is shown in Fig. 1. All members have the same cross-sectional area A and are made of a linear elastic material with Young's modulus E. All members are initially unstressed and their self-weight can be neglected. A vertical load P is then applied at joint A as shown in the figure.

(a) Find the number of redundancies. [2]

(b) Find the tensions in the bars. [19]

(c) Find the horizontal displacement at point B. [4]

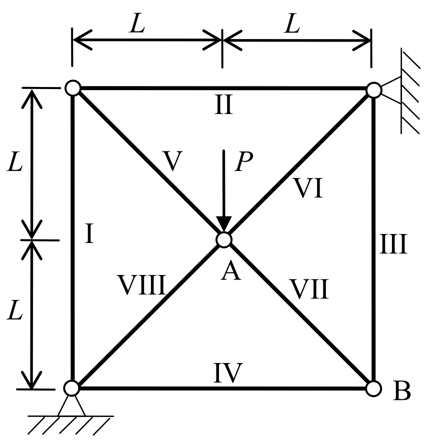

Fig. 1

2 (a) A beam with a tapered square cross-section is shown in Fig. 2(a). The beam is built in at point A and has a Young's modulus E. The square cross-section of the beam has a side-length of b at point A, and the side-length varies linearly to $b/2$ at point B. A moment M is applied at point B as shown. Use virtual work to find the rotation at point B. [8]

(b) The tapered beam in Fig. 2(a) is now fixed to a propped cantilever column as shown in Fig. 2(b). The column has a Young's modulus E and has a uniform square cross-section with side-length b.

(i) Use virtual work to show that the support reaction at point E is $R_E = 9M / 16L$ (to the left). [12]

(ii) Find the rotation at point D. [5]

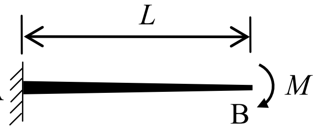

Fig. 2(a)

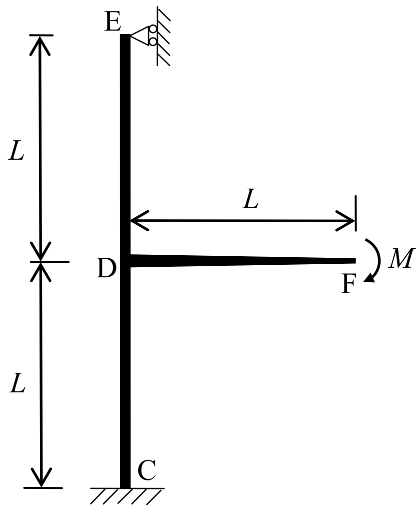

Fig. 2(b)

3 The uniform beam shown in Fig. 3 is supported by a pin at point A and a roller at point C. The supports are separated by a distance αL. The beam is loaded with a uniformly distributed load of w per unit length between points A and B, and with a point load of magnitude wL at point D. The beam has a plastic moment capacity of M_p.

(a) Assume a plastic hinge could only form at points B or C.

(i) Use upper bound analysis to calculate the value of α that minimises the M_p required to carry the specified loads. [9]

(ii) Using the value of α found in part (i), sketch the bending moment diagram. You are not required to label the diagram with values. [4]

(b) Now instead assume α = 1.8 and that only one plastic hinge could form at a distance βL from point A, where $0 < \beta < 1$.

(i) Using upper bound analysis, write an equation for the value of M_p required to carry the specified loads. [9]

(ii) List the steps you would follow to calculate the value of β that minimises the M_p required to carry the specified loads. [3]

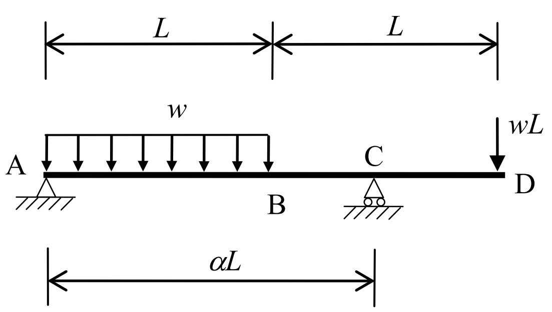

Fig. 3

SECTION B

4 The multi-span uniform beams in Fig. 4(a) and Fig. 4(b) are subjected to various uniformly distributed loads, where w is a load per unit length.

(a) For the beam in Fig. 4(a), determine the bending moment and shear force at the mid-span of segment BC. [8]

(b) For the beam in Fig. 4(b), determine the bending moment and shear force at the mid-span of segment FG. [17]

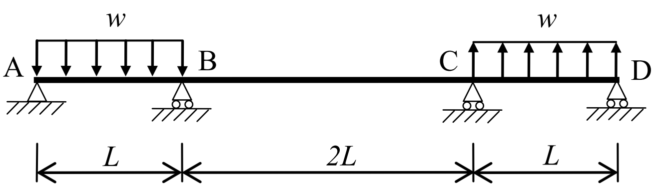

Fig. 4(a)

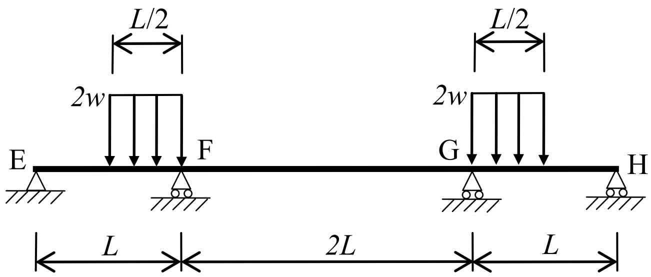

Fig. 4(b)

5 A cross section of a long dam is shown in Fig. 5(a). The dam is assumed to behave as a rigid block, and the soil as a weightless rigid-plastic continuum of uniform isotropic material with a shear yield stress k. The density of the dam is twice that of water. The ground slopes downward at 30º from point A, which is located at the bottom right corner of the dam.

(a) Initially, assume that no water is on either side of the dam. For the plane sliding block mechanism shown in Fig. 5(a) with $\phi = 30$ degrees, calculate the height h above which collapse would occur. [14]

(b) Now assume that water is added to the right hand side of the dam, as shown in Fig. 5(b). The water surface is a height $d = h/2$ above the base of the dam. Assuming a circular collapse mechanism with a centre of rotation at point A, calculate the k required to prevent collapse. Recall that the hydrostatic pressure varies linearly with depth. Assume that the underside of the dam and the sloped soil surface do not permit any seepage of water. [11]

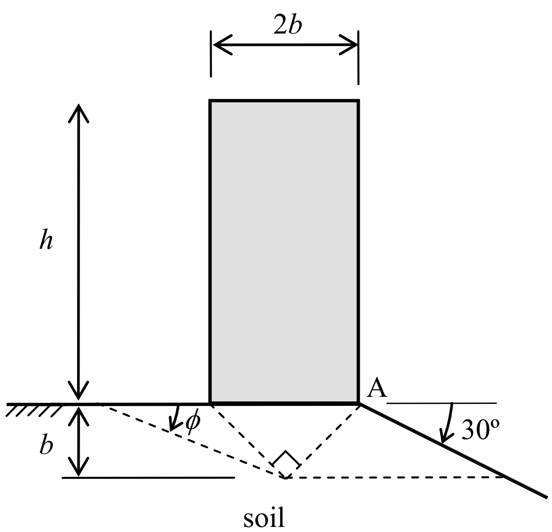

Fig. 5(a)

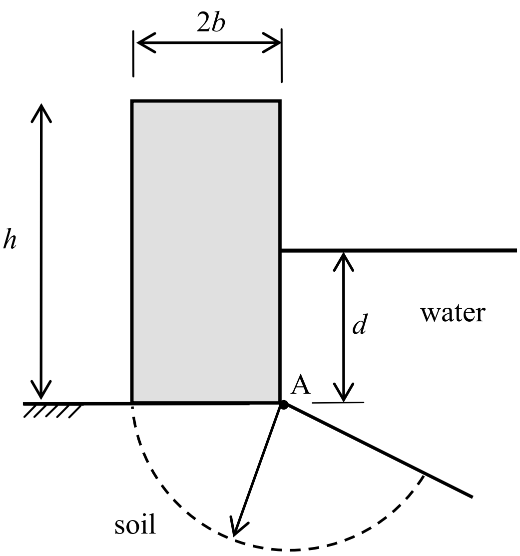

Fig. 5(b)

6 A long cylindrical steel pipe is filled with pressurised water and supported every 20 m. The pipe is continuous over the supports and has bellows at the ends to prevent longitudinal stresses from being induced by the water pressure. The pipe has a radius of r = 10 cm, a thickness of t = 1 mm, and a yield stress of σ_y = 275 MPa. The combined weight per unit length of the water and the pipe is 364 N m^{-1}.

(a) Using the Tresca yield criterion, determine the water pressure at which the top of the pipe at the mid-span would yield. [6]

(b) A strain gauge rosette has been placed on the outside of the pipe at its mid-height. The exact location and orientation of the strain gauge are not known. At a given time, the strain gauge rosette provides the following data:

$$\varepsilon_{aa} = 14\times10^{-6}, \quad \varepsilon_{bb} = 20\times10^{-6}, \quad \gamma_{ab} = 68\times10^{-6}$$

(i) Draw Mohr's circle of stress. [6]

(ii) Determine the pressure inside the pipe. [6]

(iii) Determine the distance along the pipe from the strain gauge to the nearest support, and determine a possible orientation of the strain gauge rosette with respect to the axis of the pipe. [7]

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Answers

1. (a) 2 ; (b) $\frac{P}{4}\begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \\ \sqrt{2} \\ \sqrt{2} \\ -\sqrt{2} \\ -\sqrt{2} \end{bmatrix}$; (c) $\frac{PL}{2AE}$ (to the right)

2. (a) $\frac{56LM}{b^4E}$; (b) (ii) $\frac{-15ML}{8b^4E}$

3. (a) (i) $\alpha = 1.85$; (b) (i) $M_p = wL^2\beta(-0.5\beta + 0.61)$

4. (a) $V_{centre} = \frac{-wL}{16}$, $M_{centre} = 0$; (b) $V_{centre} = 0$, $M_{centre} = \frac{9}{256}wL^2$

5. (a) $\left(2+\sqrt{3}\right)\frac{k}{\rho_w g}$; (b) $\frac{3b}{10\pi}\left(\frac{4h}{b} + \frac{1}{6}\frac{h^3}{b^3} - \frac{4}{3}\right)$

6. (a) 0.86 MPa ; (b) (ii) 102 kPa , (iii) 9.1 m , 31°