EGT1
ENGINEERING TRIPOS PART IB

Tuesday 31 May 2016 2 to 4

Paper 4

THERMOFLUID MECHANICS

*Answer not more than **four** questions.*

*Answer not more than **two** questions from each section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

*Answer not more than **two** questions from this section.*

1 The cooling system on a spacecraft consists of a heat-exchanger tube through which a coolant fluid is passed. The tube has outer radius a, wall thickness t, and thermal conductivity λ. The system is arranged such that the view factor between the tube and deep space is effectively unity. Space can be assumed to have a temperature of 0 K.

(a) There is negligible resistance to heat transfer between the bulk of the fluid and the inner surface of the tube. Show that the local heat flow per unit length of tube is given by

$$\dot{Q} = \frac{2\pi\lambda\left(T_s - T_f\right)}{\ln\left(1 - \frac{t}{a}\right)},$$

where T_s and T_f are the temperatures of the tube outer surface and the fluid respectively. [5]

(b) By considering the rate at which heat must leave the surface and be radiated into space, show that, for a thin-walled tube (i.e. $t \ll a$),

$$\dot{Q}\left(\frac{t}{2\pi a\lambda} + \frac{1}{2\pi a\varepsilon\sigma T_s^3}\right) = T_f,$$

where ε is the emissivity of the tube surface and σ is the Stefan-Boltzmann constant. [5]

(c) The coolant fluid has a heat capacity of 1 kJ kg^{-1} K^{-1}, and enters the tube with temperature 320 K at a flow rate of 0.001 kg s^{-1}. The tube is 5 m long, with outer radius 1 cm and wall thickness 1 mm. It is made of material with thermal conductivity 200 W m^{-1} K^{-1}, and its outer surface can be taken as black.

(i) Show that the thermal resistance of the tube wall can be neglected. [5]

(ii) Calculate the temperature of the fluid leaving the tube. [8]

(iii) What is the total rate at which heat is lost to space? [2]

2 A cooling system uses a vapour-compression cycle with water as the working fluid. Saturated steam at 50 °C leaves the evaporator and is then compressed to 0.5 bar. The compressor is adiabatic with an isentropic efficiency of 0.9, and the mass flow through the compressor is 1 kg s^{-1}. The steam is then condensed isobarically to a saturated liquid and finally returned to the evaporator via an adiabatic throttle valve. The evaporator can be assumed to be at constant pressure.

(a) Sketch the cycle on an h-s diagram. [3]

(b) Determine:

(i) the pressure in the evaporator and the lowest temperature at which heat is rejected; [2]

(ii) the specific enthalpy of the fluid leaving the compressor; [4]

(iii) the dryness fraction of the fluid leaving the throttle valve; [3]

(iv) the coefficient of performance of the system. [3]

(c) The work for the vapour-compression cycle is to be generated by combining it with an ideal (open) Rankine cycle. A fraction of the water leaving the condenser is diverted, pumped to 20 bar and then heated, at constant pressure, to 500 °C. The vapour is passed through an adiabatic and reversible turbine, leaving at a pressure equal to that of the evaporator. The fluid leaving the turbine is returned to the vapour-compression cycle by mixing it with the fluid leaving the throttle valve. Pump work can be neglected.

(i) What fraction of the mass flow of water through the compressor must be diverted through the Rankine cycle to ensure that no additional external work is required by the compressor? [4]

(ii) Determine the heat absorbed by the evaporator, and hence define a suitable metric for the performance of the combined system. Comment on the magnitude of your metric. [6]

3 A gas-turbine cycle consists of an isentropic compressor with a pressure ratio of 10, a heater, and a turbine with an isentropic efficiency of 0.9. The turbine and compressor are both adiabatic. There is no pressure drop in the heater. Air enters the compressor at environmental conditions: 25 °C and 1 bar. It leaves the heater at 1200 K and, after passing through the turbine, exhausts to the environment. The air can be assumed to behave as a perfect gas throughout.

(a) (i) Define the availability, b, in terms of other thermodynamic state variables and the environmental temperature. [2]

(ii) Starting from the appropriate forms of the first and second laws, show that, for a steady-flow system operating in an environment with temperature T_0,

$$\Delta b = -w_x + \int \left(1 - \frac{T_0}{T}\right) \mathrm{d}q - T_0 \Delta s_{irrev},$$

and explain briefly the physical meaning of the terms on the right-hand side. [3]

(b) Calculate (per kg of air):

(i) the power potential lost due to irreversibility in the compressor and in the turbine; [5]

(ii) the transfer of power potential to the air in the heater; [3]

(iii) the first- and second-law efficiencies of the cycle. [3]

(c) The system is modified so that, instead of heat being provided between the compressor exit and the turbine inlet, it is removed, reducing the turbine inlet temperature to 308 K.

(i) Determine the temperature of the air leaving the turbine, the power potential lost to irreversibility, and the net work input. [3]

(ii) Calculate the power potential of the exhaust gas. Explain why it is positive, and how it could be extracted. [6]

SECTION B

*Answer not more than **two** questions from this section.*

4 (a) The equation governing steady, two-dimensional, parallel-streamline flow of an incompressible viscous fluid is

$$\mu \frac{\mathrm{d}^2 u}{\mathrm{d}y^2} = \frac{\mathrm{d}p}{\mathrm{d}x},$$

where μ is the fluid's dynamic viscosity, u its velocity, y the streamline-normal coordinate, and $\mathrm{d}p/\mathrm{d}x$ the streamwise pressure gradient.

(i) Explain briefly how this equation is derived. [6]

(ii) Find the velocity profile $u(y)$ for a channel with walls at $y = \pm h$. [4]

(iii) What is the volumetric flow rate (per unit depth perpendicular to the flow plane) in the channel of part (ii)? [3]

(b) An incompressible viscous fluid flows along a two-dimensional channel of linearly varying width $2h(x)$, as shown in Fig. 1. The dynamic viscosity is μ and the volumetric flow rate Q. An engineer argues that the variation in width is sufficiently gradual that, at each streamwise position x, the results of part (a) apply. Subject to this assumption, find the pressure drop over the length AB of the channel. [6]

(c) The full equation for the flow of part (b) contains a term $\rho u \partial u/\partial x$ (where ρ is the fluid's density). The engineer seeks an order-of-magnitude condition for it to be negligible relative to the contributions recognised in part (a). By first expressing u as a function of Q, y, and h, or otherwise, find this condition. (Your answer should be presented in terms of $\mathrm{d}h/\mathrm{d}x$ and the Reynolds number $\rho Q/\mu$.) [6]

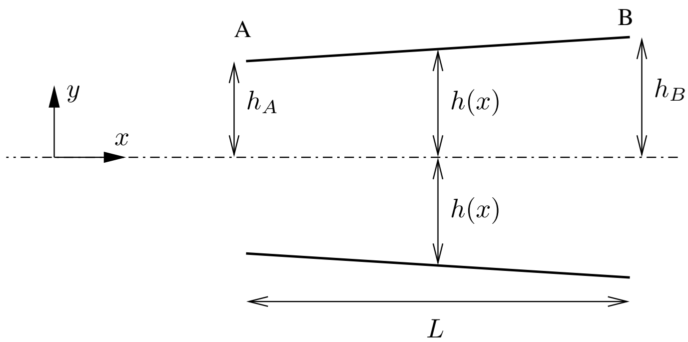

Fig. 1

5 (a) A pump supplies water of density ρ from a reservoir to a fountain through a wide pipe and a smooth nozzle, as shown in Fig. 2. Losses in the flow are negligible, and the streamlines at the nozzle exit are parallel. The fountain rises to a height H above the reservoir surface.

(i) What increase in total pressure must be provided by the pump? [4]

(ii) Given that the nozzle exit area is A, what is the volumetric flow rate? [4]

(b) The total-pressure rise Δp_T across a pump is known to be a function of: rotational speed, N; impeller diameter, D; volumetric flow rate, Q; fluid density, ρ; and fluid dynamic viscosity, μ.

(i) Derive the general form of this function in terms of dimensionless parameters. [4]

(ii) If the influence of viscosity is negligible, show that your expression from (i) can be written as

$$\frac{\Delta p_T}{\rho(ND)^2} = f\left(\frac{Q}{ND^3}\right). \quad [2]$$

(c) The pump of part (a) is sized to provide the required fountain height and flow rate when its efficiency (defined as the ratio of output flow power to input electrical power) is best. However, the fountain's owners find it too expensive to run, and decide to sacrifice flow rate (by reducing the nozzle area) while maintaining the same height.

(i) Explain why the pump speed must also be altered. (You can assume that viscous effects are negligible.) [2]

(ii) Fig. 3 shows the dimensionless pump characteristic and the associated values of efficiency, η. By what factors would the efficiency, speed, and flow rate change if Q/ND^3 were reduced from its design value of 0.1 to 0.05? [5]

(iii) Discuss the extent to which a reduction in flow rate, as proposed, will translate to lower electricity costs. [4]

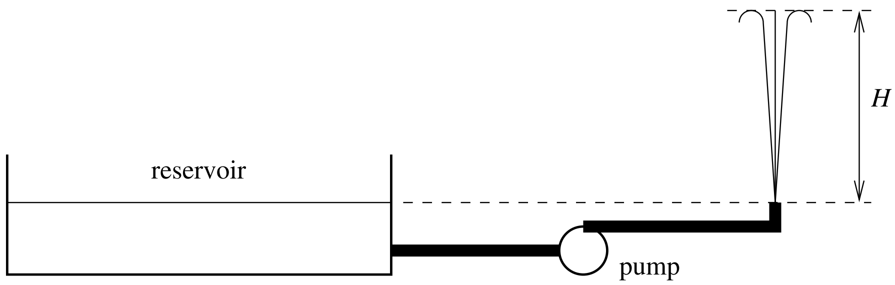
reservoir
pump
H

Fig. 2

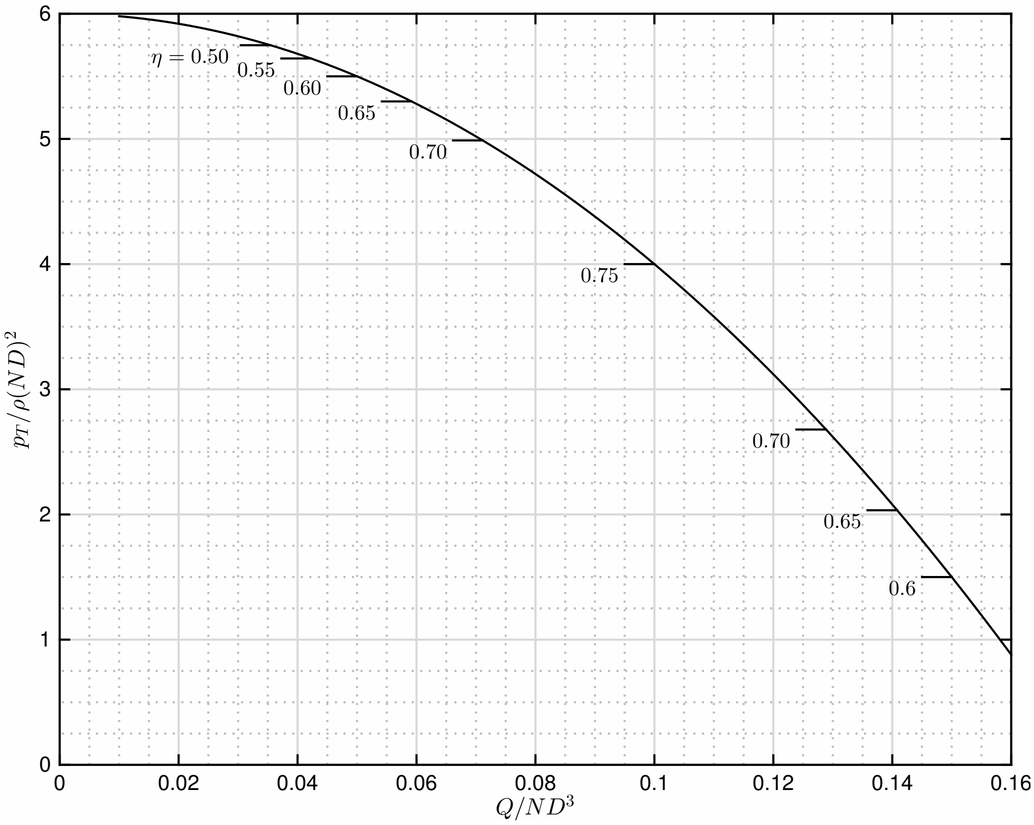
$\eta = 0.50$
0.55
0.60
0.65
0.70
0.75
0.70
0.65
0.6
$p_T/\rho(ND)^2$
Q/ND^3
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
1
2
3
4
5
6

Fig. 3

6 (a) State the equation for the pressure gradient perpendicular to the streamlines in a steady, inviscid fluid flow, providing definitions of all the symbols that you use. Also give a physical interpretation of this equation and its component terms. [6]

(b) Water flows inviscidly in an open channel whose cross-section is rectangular and whose bottom surface is horizontal. The channel is originally straight, and then bends in a circular arc, as shown in Fig. 4. The arc is centred on the point O.

(i) Explain why, in the bend, the fluid depth must increase with distance r from O. [4]

(ii) Well into the bend, the fluid streamlines form horizontal, concentric circular arcs, and the depth, h, is given by

$$h = \frac{r^2}{H},$$

where H is a constant. What is the fluid velocity? [3]

(iii) What is the total pressure, relative to atmospheric, of the flow well into the bend? (Take the datum for vertical position as the bottom surface of the channel.) [2]

(iv) How are the flow conditions in the bend linked to those upstream? In the light of your answer, sketch the *approximate* distribution of fluid velocity across the straight part of the channel. [5]

(c) In a real flow, boundary layers form on solid surfaces in contact with the fluid. Discuss the implication of the associated flow retardation for radial sediment transport in a river bend. [5]

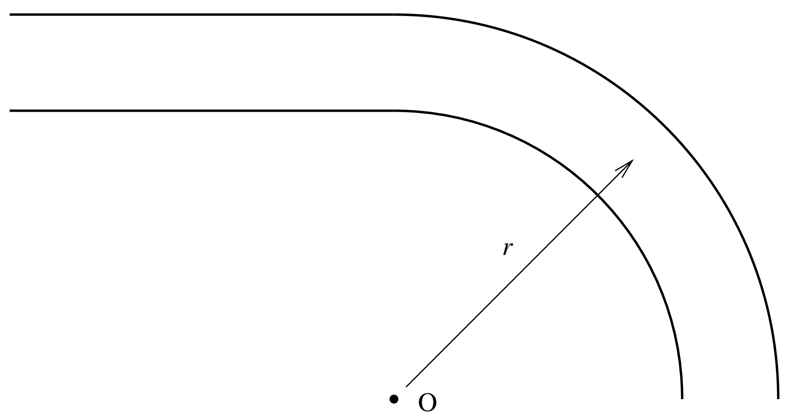

Fig. 4

END OF PAPER

ENGINEERING TRIPOS PART IB

Paper 4: THERMOFLUID MECHANICS

ANSWERS

1(c) (ii) 228K (iii) 92W

2(b) (i) 12.35kPa, 81.3°C (ii) 2867kJ/kg (iii) 0.055 (iv) 8.18

(c) (i) 0.254 (ii) 1732kJ/kg

3(a) (i) $b = h - T_0 s$

(b) (i) 0, 26.5kJ/kg (ii) 408kJ/kg (iii) 39%, 60%

(c) (i) 174K, 26.5kJ/kg, 144kJ/kg (ii) 36.5kJ/kg

4(a) (ii) $u = \dfrac{h^2 - y^2}{2\mu}\left(-\dfrac{\mathrm{d}p}{\mathrm{d}x}\right)$ (iii) $\dfrac{2h^3}{3\mu}\left(-\dfrac{\mathrm{d}p}{\mathrm{d}x}\right)$

(b) $\dfrac{3\mu QL}{4}\dfrac{h_A + h_B}{h_A^2 h_B^2}$; alternatively $\dfrac{3\mu QL}{4\left(h_B - h_A\right)}\left(\dfrac{1}{h_A^2} - \dfrac{1}{h_B^2}\right)$

(c) $\dfrac{\rho Q}{\mu}\dfrac{\mathrm{d}h}{\mathrm{d}x} << 1$

5(a) (i) $\rho g H$ (ii) $A\sqrt{2gH}$

(b) (i) $\dfrac{\Delta p_T}{\rho\left(ND\right)^2} = f\left(\dfrac{Q}{ND^3}, \dfrac{\mu}{\rho ND^2}\right)$, or equivalent

(c) (ii) 0.8, 0.853, 0.426

6(b) (ii) $r\sqrt{2g/H}$ (iii) $2\rho g r^2/H$