Version JL/5

EGT1
ENGINEERING TRIPOS PART IB

Thursday 2 June 2016 2 to 4

Paper 6

INFORMATION ENGINEERING

Answer not more than ***four*** *questions.*

Answer not more than ***two*** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper, graph paper, semilog graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

*Answer not more than **two** questions from this section.*

1 Consider a car moving along a straight line. Car position, velocity and acceleration at time t are denoted by $y(t)$, $\dot{y}(t)$ and $\ddot{y}(t)$, respectively. The position and velocity are zero at $t = 0$. The dynamics of the car satisfy the balance of forces $m\ddot{y}(t) + c\dot{y}(t) = u(t)$ where $m > 0$ is the mass of the car, $c > 0$ is the damping coefficient and $u(t)$ is the force exerted on the car by the engine.

(a) (i) Show that the transfer function $G(s)$ from $\bar{u}(s)$ to $\bar{y}(s)$ is given by

$$G(s) = \frac{\frac{1}{m}}{s(s+\frac{c}{m})}$$

[$\bar{f}(s) \equiv \mathscr{L}(f)$ denotes the Laplace transform of f]. [3]

(ii) Show that the impulse response is given by $g(t) = \frac{1}{c}\left(1 - e^{-\frac{c}{m}t}\right)$ for $t \geq 0$. Sketch $g(t)$. [7]

(iii) Show that the system is marginally stable by finding $A > 0$ and $B > 0$ such that

$$\int_0^T |g(t)|dt < A + BT \qquad \text{for all } T > 0$$

[5]

(b) A simple PI (proportional-integral) cruise-control system is shown in Fig. 1. For $m = 1$ and $c = 1$, the closed-loop transfer function $T(s)$ from $\bar{r}(s)$ to $\bar{z}(s) = \mathscr{L}(\dot{y})$

$$T(s) = \frac{k_p s + k_i}{s^2 + (1+k_p)s + k_i}$$

(i) Take $k_i = \frac{(1+k_p)^2}{4}$ and let $k_p > 0$. What are the closed loop pole locations? [3]

(ii) For $k_p = 0$, find $k_i > \frac{1}{4}$ to achieve a damping ratio equal to 0.5. Sketch the resulting impulse response. [7]

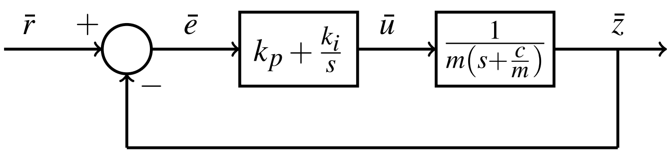

Fig. 1

2 An industrial process is modelled by the transfer function

$$G(s) = \frac{k}{s}\mathrm{e}^{-Ds}$$

where $D \geq 0$ is a transport delay and $k = \frac{1}{2}$.

(a) (i) For $D = 0$, sketch the Bode and Nyquist diagrams of $G(s)$. Find the gain margin. [5]

(ii) Discuss the effect of a delay $D > 0$ on the Nyquist locus. [4]

(iii) Find the excitation frequency ω at which the input and output have the same amplitude. [3]

(iv) A constant unit input is applied to the process. Show that the output grows without bound. [3]

(b) To improve the steady-state response at constant inputs, consider the control loop in Fig. 2. For simplicity take $D = 0$ in $G(s)$ and take $K(s) = k_p > 0$.

(i) Find, in terms of $G(s)$ and $K(s)$, the two closed loop transfer functions relating: $\bar{r}(s)$ to $\bar{y}(s)$ and $\bar{d}(s)$ to $\bar{y}(s)$. [5]

(ii) Constant unit inputs are applied to the controlled process: $r(t) = 1$ and $d(t) = 1$. Compute $y(t)$ at steady state. [5]

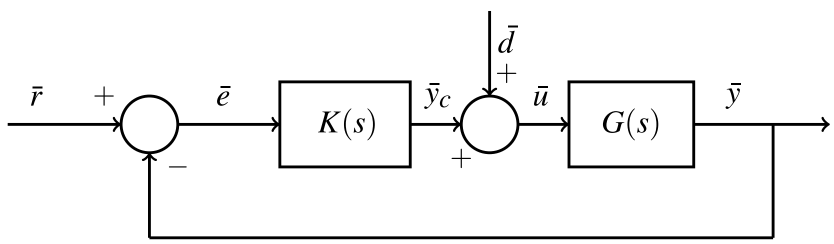

Fig. 2

3 The two tank system in Fig. 3 is part of a water treatment plant. The pump controls the inflow of fluid q_0 with the goal of stabilising the tank levels at values of h_1^e and h_2^e. Near equilibrium, levels h_1, h_2 are related to flows q_0, q_1 and q_2 by the equations

$$\dot{h}_1 = q_0 - q_1 \qquad \dot{h}_2 = q_1 - q_2$$

To a first order approximation the valve behaviour is governed by

$$q_1 = \alpha(h_1 - h_2) \qquad q_2 = \beta h_2$$

where $\alpha > 0$ and $\beta > 0$.

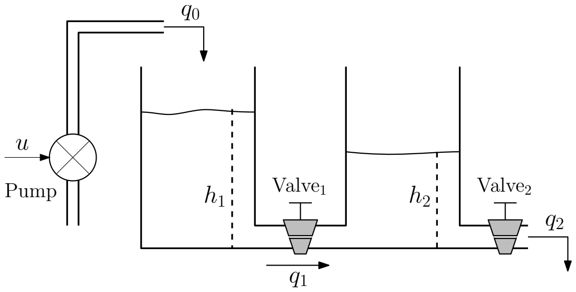

Fig. 3

(a) Show that the plant transfer function relating $\bar{q}_0(s)$ to $\bar{q}_2(s)$ is given by

$$G(s) = \frac{\alpha\beta}{s^2 + (2\alpha + \beta)s + \alpha\beta}$$

[9]

(b) Sketch the block diagram of the closed loop system given by the plant $G(s)$, the pump controller with transfer function $K(s)$ relating $\bar{q}_0(s)$ to $\bar{u}(s)$, a disturbance d (leak in the pipe after the second valve), and unity negative feedback $u = -y$, where $y = q_2 + d$. [7]

(c) For $\alpha = 2$ and $\beta = 4$, the company adopts the controller $K(s) = \frac{k}{s}$, $k = 1$, which guarantees asymptotic stability of the closed loop.

(i) Estimate gain and phase margins from the Bode diagram in Fig. 4. [3]

(ii) Discuss the effects of a larger/smaller gain k on the phase and gain margins. [2]

(iii) Show that $y(t) = 0$ at steady state, given a constant leak $d(t) = 1$. [4]

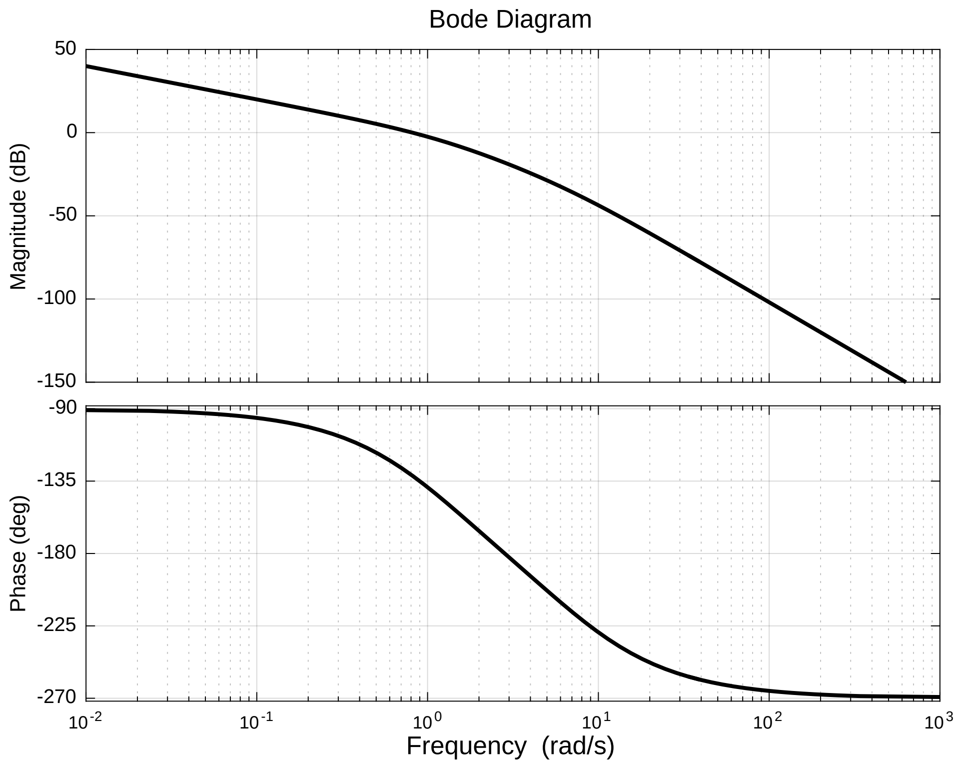

Fig. 4

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SECTION B

Answer not more than ***two*** *questions from this section.*

4 (a) If the Fourier transform of $x(t)$ is $X(\omega)$, derive the Fourier transform of the shifted function $x(t-T)$. [4]

(b) The Fourier transform of a triangular pulse centred on the origin with height a and width $2b$ is $ab\ \mathrm{sinc}^2\left(\frac{\omega b}{2}\right)$. Now consider the signal shown in Fig. 5, where $x(t)=0$ for $|t|>T$.

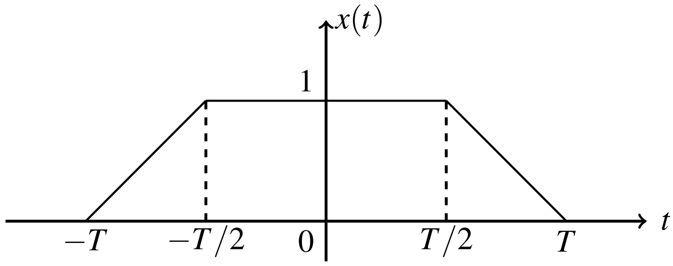

Fig. 5

By expressing $x(t)$ as a sum of three triangular pulses, or otherwise, show that its Fourier transform, $X(\omega)$, can be written as

$$X(\omega) = f(T,\omega)\ \mathrm{sinc}^2\left(\frac{\omega T}{4}\right)$$

and find $f(T,\omega)$. [6]

(c) By considering the signal of part (b) as the difference of two triangular pulses, show that an alternative expression for $X(\omega)$ is

$$X(\omega) = 2T\ \mathrm{sinc}^2\left(\frac{\omega T}{2}\right) - \frac{T}{2}\ \mathrm{sinc}^2\left(\frac{\omega T}{4}\right)$$

[5]

(d) Sketch $X(\omega)$ and determine the width of the mainlobe (take this to be the distance between the first negative and positive zeroes). [5]

(e) Using Parseval's Theorem or otherwise, approximate the energy in the mainlobe of $X(\omega)$. [Note: take *energy* to be given by $\frac{1}{2\pi}\int_{-\infty}^{+\infty}|X(\omega)|^2 d\omega$]. [5]

5 (a) $x(t)$ is a bandlimited signal. Its spectrum, $X(\omega)$, satisfies $X(\omega) = 0$ for $|\omega| \geq \omega_c$.

(i) If we sample $x(t)$ by the pulse train $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$ what value should the sampling period, T, be set at so as to just avoid aliasing artefacts when we reconstruct the original signal from its samples. [3]

(ii) Suppose we now take a **finite** set of N samples from our signal which has been sampled at period T. Let these be $\{x_0, x_1, ..., x_{N-1}\}$. The discrete Fourier Transform (DFT) of this sequence is given by $\{X_o, X_1, ..., X_{N-1}\}$, where

$$X_k = \sum_{n=0}^{N-1} x_n \mathrm{e}^{-jkn2\pi/N} \quad \text{for} \quad 0 \leq k \leq N-1$$

Show that the inverse DFT is given by

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k \mathrm{e}^{jkn2\pi/N} \quad \text{for} \quad 0 \leq n \leq N-1$$

[9]

(b) An information signal $m(t)$ is transmitted using *Double Sideband Suppressed Carrier* (DSB-SC) modulation as

$$s(t) = m(t)\cos(2\pi f_c t)$$

where f_c is the carrier-frequency. The transmitted waveform is attenuated by the channel, and the receiver gets

$$y(t) = \alpha s(t)$$

The attenuation factor $\alpha < 1$ is known to the receiver.

(i) Draw a block diagram of the receiver used to recover $m(t)$ from $y(t)$ assuming that $m(t)$ is a baseband signal with bandwidth W, with $W \ll f_c$. Specify the cut-off frequencies and gain of any filter used. You may assume that the receiver has a perfect copy of the carrier wave available. [8]

(ii) Now suppose that, unknown to the receiver, its version of the carrier wave has a phase lag of 45° from the transmitter's carrier. If this carrier is used in the receiver of part (b)(i), what is the recovered signal? [5]

6 (a) A communication link carries digital data modulated using Pulse Amplitude Modulation (PAM), where the information symbols $X_1, X_2, \ldots$ are drawn from the 3-point constellation $\{-2A, 0, 2A\}$. After demodulation at the receiver, the discrete-time sequence at the output of the demodulator is

$$Y_k = X_k + N_k \qquad \text{for } k \geq 0$$

where N_k is additive Gaussian noise with mean zero and variance σ^2.

(i) What is the optimal detection rule, assuming that the three constellation symbols are equally likely? [3]

(ii) Compute the probability of detection error for each of the symbols, and hence show that the overall probability of detection error P_e is given by

$$P_e = \frac{4}{3} Q\left(\frac{A}{\sigma}\right)$$

where $Q(y) = 1 - \Phi(y)$, and $\Phi(y)$ is the Gaussian cumulative distribution function. [6]

(iii) Compute E_s, the average energy per constellation symbol, and express the detection error probability in part (a)(ii) in terms of the signal-to-noise ratio $\frac{E_s}{\sigma^2}$. [3]

(iv) Using the approximation $Q(x) \approx \frac{1}{2} e^{-x^2/2}$ for $x \geq 0$, find the value of $\frac{E_s}{\sigma^2}$ needed for a detection error probability of 10^{-5}. Express your answer in dB. [3]

(b) Consider digital information encoded by a $(7,4)$ Hamming code, where 3 parity-check bits are appended to each block of 4 data bits. Denoting the data bits by s_1, s_2, s_3, s_4, recall that the 7-bit Hamming codeword is given by

$$[c_1,\ c_2,\ c_3,\ c_4, c_5, c_6,\ c_7] = [s_1,\ s_2,\ s_3,\ s_4,\ s_1 \oplus s_2 \oplus s_3,\ s_2 \oplus s_3 \oplus s_4,\ s_1 \oplus s_3 \oplus s_4]$$

where $\oplus$ denotes modulo-two addition.

(i) Suppose that a codeword is transmitted over a binary symmetric channel (BSC), and the received sequence is $\mathbf{r} = [0,\ 1,\ 1,\ 1,\ 1,\ 0,\ 0]$. Decode the received sequence to a codeword. [5]

(ii) With a Hamming code, a decoding error occurs when the channel flips two or more bits. Compute the probability of decoding error when a 7-bit Hamming codeword is transmitted over a BSC with a crossover probability of 0.1. [5]

END OF PAPER