Version CYB/4

EGT1
ENGINEERING TRIPOS PART IB

Friday 3 June 2016 9 to 11.30
9 to 10.30 Foreign Language Option

Paper 8

SELECTED TOPICS

Answer ***one*** *question from Section A. In addition:*

If you are not taking the Foreign Language option, answer ***four*** *questions, taken from only two of sections B–H. Not more than* ***two*** *questions from each section may be answered.*

If you are taking the Foreign Language option, answer ***two*** *questions from one of Sections B–H.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

Answers to questions in each section should be tied together and handed in separately.

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed

Attachments: Data Sheets for Section B (6 pages) and Section E (2 pages)

Supplementary page: one extra copy of Fig. 3 (Question 6)

Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A: ***Introductory Business Economics***

Answer not more than ***one*** *question from this section.*

1 (a) Consider the market for wine. Draw a diagram representing conventionally shaped demand and supply curves. Represent and explain changes in the market equilibrium following:

(i) an increase in consumer income;

(ii) the adoption of more efficient bottling equipment among producers;

(iii) a fall in the price of beer;

(iv) a poor harvest due to bad weather. [5]

(b) With reference to the problem of resource allocation:

(i) Why is perfect competition generally considered to be the most efficient market structure? [5]

(ii) If perfect competition is the most efficient market structure, why is it so rare in real markets? [5]

(c) Define the components of national income. Identify and explain instances when specific injections and withdrawals might not be in equilibrium. [10]

2 (a) Define a firm's fixed and variable costs. By means of a diagram or diagrams represent and explain the relevant (conventionally shaped) total, average and marginal costs curves in the short run. [5]

(b) Consider an oligopolistic market structure.

(i) Compare and contrast the Cournot and Bertrand models of oligopoly. [5]

(ii) Does competition among oligopolists maximise total industry profit? Explain your answer. [5]

(c) Illustrate the fundamental principles of Keynesian consumption theory. [10]

SECTION B: ***Civil and Structural Engineering***

*Answer not more than **two** questions from this section.*
Note Data Sheet at end of the paper.

3 (a) A construction is planned on fully saturated clayey soil. The soil has the following geotechnical properties: Specific gravity $G_s = 2.70$, void ratio $e = 1.20$, friction angle $\phi' = 25°$ and undrained shear strength $c_u = 50$ kNm^{-2}.

(i) Evaluate the water content w and saturated unit weight γ (in kNm^{-3}). [3]

(ii) At a depth of 10 metres, what are the total vertical and effective stresses? Assume that the water table is 1 metre below the ground surface and the soil is fully saturated to the ground level. [3]

(iii) For the soil at 10 m depth, if the vertical total stress is kept constant but the horizontal total stress is increased, estimate the horizontal total stress required to fail the soil in undrained conditions. [4]

(iv) For the same conditions given in (iii), estimate the horizontal total stress required to fail the soil in drained conditions. [4]

(b) To determine the safety of a tunnel constructed in clay, a normalised design parameter called *stability ratio* is used.

(i) Define the stability ratio in terms of vertical total stress, temporary support pressure and undrained shear strength of the clay. [2]

(ii) A tunnel is to be constructed in clayey ground with an undrained shear strength $c_u = 100$ kNm^{-2} and a unit weight of 20 kNm^{-3}. Estimate the depth of the tunnel that can be safely excavated in open face mode. [2]

(iii) The tunnel goes under some important buildings and compensation grouting is proposed to mitigate the effect of tunnelling on building damage. Using diagrams, explain the construction and execution processes of a compensation grouting system. Describe how a *tube a'manchette* (TAM) works. [4]

(iv) Even when compensation grouting is successfully conducted during tunnel construction, the client is worried about the long-term effect. Why is this so? [3]

4 A retaining wall of 15 m length is constructed in silty sand underlain by an impermeable rock. An excavation is then made on one side of the wall to a depth of 10 m as shown in Fig. 1. A temporary prop is placed at the top level of the wall during construction to resist the difference in earth pressures. The silty sand has a saturated unit weight of $\gamma = 17$ kNm^{-3} and a critical state friction angle of $\phi'_{crit} = 30°$. The soil is in drained conditions during the construction and the wall moves sufficiently to mobilise active pressures fully on the soil side. The lateral earth pressure coefficient K_{mob} on the excavation side is considered to be a constant value. It is assumed that the wall is relatively impermeable compared to the silty sand during the construction stage.

(a) If the water tables on both sides of the wall are at the original ground level, answer the following questions.

(i) Compute the horizontal total and effective stress profiles along the two sides of the wall. Sketch the profiles marking the salient values. [6]

(ii) Compute the value of K_{mob} and evaluate the ratio K_p/K_{mob}, where K_p is the passive earth pressure coefficient. [5]

(iii) Evaluate the prop force P. [5]

(b) The water level on the excavation side is now at the excavation level, whereas that on the soil side remains at the original ground level.

(i) Evaluate the horizontal total and effective stress profiles along the two sides of the wall and show that the excavation is not safe. [6]

(ii) Discuss possible engineering solutions that may allow safe excavation. [3]

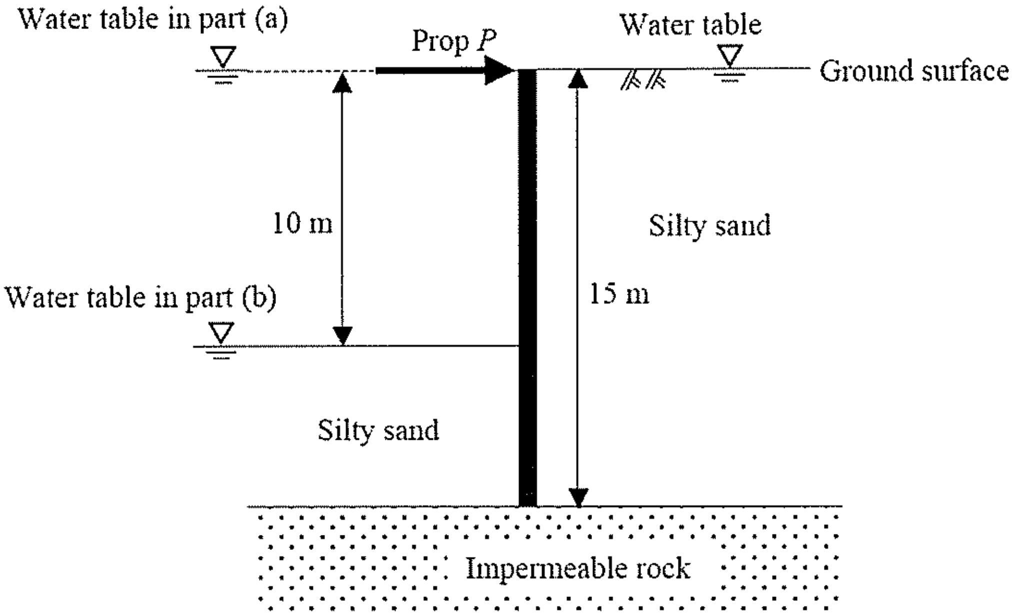

Fig. 1

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5 An excavation for a new underground station is being built using diaphragm walls that are 18 m deep. The critical design case for the walls is a temporary condition, when the base slab has been placed and a closely spaced row of props has been inserted at a depth of 8 m. The situation is illustrated in Fig. 2. For this case, the loading can be regarded as a triangular distribution of forces, which includes the hydrostatic and the soil loads.

(a) Sketch the bending moment and shear force diagrams for the wall marking salient values. [6]

(b) The wall is built from concrete with a cube strength of 40 N mm^{-2}. Determine the thickness of the wall that is required if it is to be singly reinforced using high yield steel of yield strength 460 N mm^{-2}. Also, calculate the amount of reinforcement that is required at the propping position and where the bending moment is a maximum between the prop and the base. [9]

(c) Discuss qualitatively whether there would be any economic advantage in making the wall doubly reinforced. [5]

(d) The props are eventually to be replaced by a cast slab. The reinforcement of the slab is to be joined to the main wall reinforcement by means of screw-in couplers at the face. Suggest a layout of the reinforcement in the wall at this position that satisfies the requirements found in part (b) above that will facilitate this later work. [5]

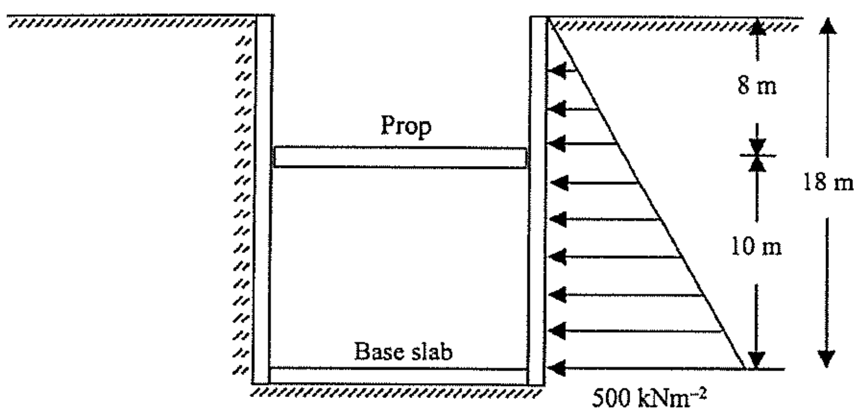

Fig. 2

SECTION C: ***Mechanics, Materials and Design***

Answer not more than ***two*** *questions from this section.*

6 (a) Discuss noise and vibration issues as related to wind turbines. [7]

(b) Figure 3 gives part of the time-varying stress pattern suffered by the critical part of a wind-turbine blade. Use this figure to identify the half-cycles of stress to be used in a fatigue analysis. Outline how such a fatigue analysis could proceed. *An additional copy of Fig. 3 is attached to the back of this paper. It should be detached and handed in with your answers.* [9]

(c) Consider a wind turbine blade design which scales with the length L of the blade. Use a simple model of the blade geometry and loading to derive scaling laws for the effect of L on the root bending stresses associated with:

(i) self-weight loading and

(ii) aerodynamic loading.

Identify carefully any assumptions you make. [9]

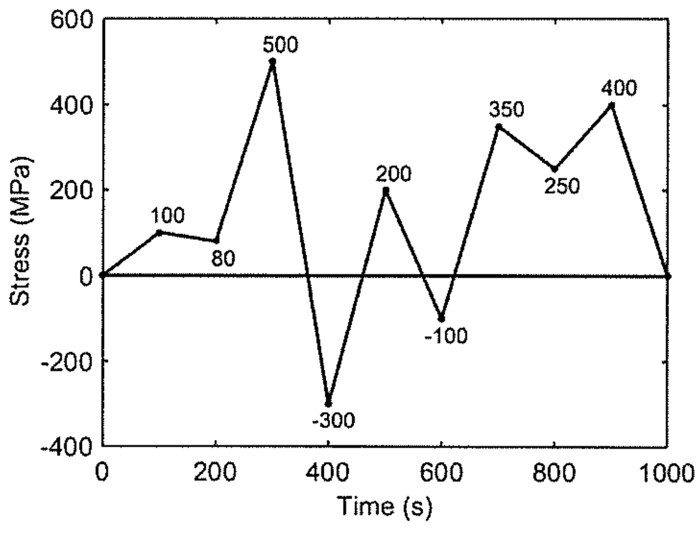

Fig. 3

7 (a) A fixed-speed 1.2 MW wind turbine generator produces rated output power at its rated wind speed of 12 m s^{-1}. At the most probable wind speed of 6 m s^{-1} the system operates at its optimum tip-speed ratio of 8, at which the power coefficient is 0.4. Assume that the power coefficient is proportional to the tip-speed ratio at other wind speeds that are greater than 6 m s^{-1}. The turbine drives a three-phase, star-connected, 16-pole cage rotor induction generator via a gearbox. The generator has its stator windings connected directly to the 50 Hz, three-phase 6.6 kV grid. Take the density of air to be 1.23 kg m^{-3}.

(i) Find the tip-speed ratio and hence power coefficient when operating at rated wind speed, and hence determine the turbine diameter. [5]

(ii) Find the generator speed and hence find the gearbox ratio such that the system operates at its optimum tip-speed ratio at the most probable wind speed of 6 m s^{-1}. [3]

(b) The cage rotor induction generator has the following equivalent circuit parameters: $R_1 = R_2' = 1\ \Omega$; $X_1 = X_2' = 2\ \Omega$; X_m and R_0 are large enough to be ignored. Find the torque, slip and phase current of the generator when the system is operating at:

(i) the rated wind speed; [5]

(ii) the most probable wind speed. [3]

The following may be quoted without proof: $P = \frac{1}{2}C_p \rho A v^3$, $\lambda = \frac{\omega R}{v}$, $T = \frac{3V^2 s}{\omega_s R_2'}$.

(c) For a number of years, the average growth rate in global installed capacity of wind power has been approximately 40 GW. A sustainability assessment aims to consider the implications of this growth on the consumption of key materials used in wind turbines.

(i) Use the data in Table 1 to estimate the global consumption rate of neodymium in wind turbines, as a percentage of the current annual production of this element. The approximate consumption of neodymium per unit of installed wind power is 0.012 kg kW^{-1}. [1]

(ii) Explain briefly what is meant by a *critical material*. Why is neodymium classified by the USA and the EU as a critical material? Is the consumption rate in wind turbines significant? [4]

(iii) The annual global consumption of CFRP in wind turbines is estimated to be 6,700 tonnes. Total CFRP consumption is currently 45% of global capacity, for which the data are given in Table 2. Comment on any strategic differences between neodymium and CFRP in relation to material supply for the wind turbine market. [4]

Table 1

	Annual production rate of neodymium by nation (tonnes/year)
China	19,500
India	450
Others	70

Table 2

	Annual production capacity of CFRP by nation ($\times$1000 tonnes/year)
Japan	20.7
USA	31.1
Taiwan	8.8
EU	22.0
Others	22.0

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8 A three-bladed wind turbine has been stopped for maintenance by a mechanical brake. It is restarted from stationary ($\omega = 0$) by releasing the brake when the turbine is pointing directly into a wind with constant speed $V = 8$ m s^{-1} and air density ρ of 1.2 kg m^{-3}. The 20 m long turbine blades are of fixed pitch, and have a local twist angle θ, a chord length c (see Fig. 4) and a mass per unit length m, all of which vary with radius r as listed in Table 3. The blade has a uniform aerofoil shape with coefficients of lift and drag plotted as a function of angle of attack in Fig. 5.

(a) Qualitatively explain the shape of the lift and drag coefficient curves shown in Fig. 5. [3]

(b) Calculate the initial torque acting on the turbine, detailing any assumptions. [11]

(c) Hence calculate the initial angular acceleration $\dot{\omega}$. [5]

(d) Estimate how long the turbine will take to reach its operating speed of $\omega = 30$ rpm. [3]

(e) Briefly explain how a more accurate answer for part (d) could be obtained, and what additional information and assumptions would be needed. [3]

Table 3

Radius r (m)	2	6	10	14	18
Twist θ (degrees)	20	13	5	2	0.5
Chord c (m)	1.6	1.5	1.3	1.0	0.7
Mass per unit length m (kg m^{-1})	130	110	85	50	25

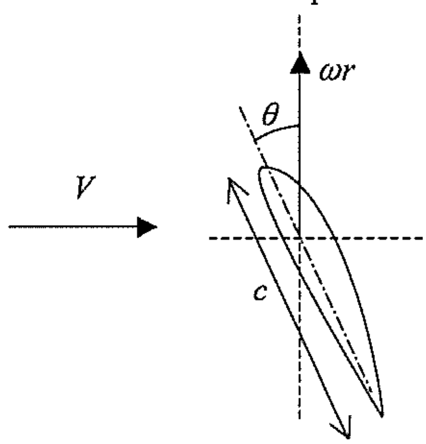

Fig. 4

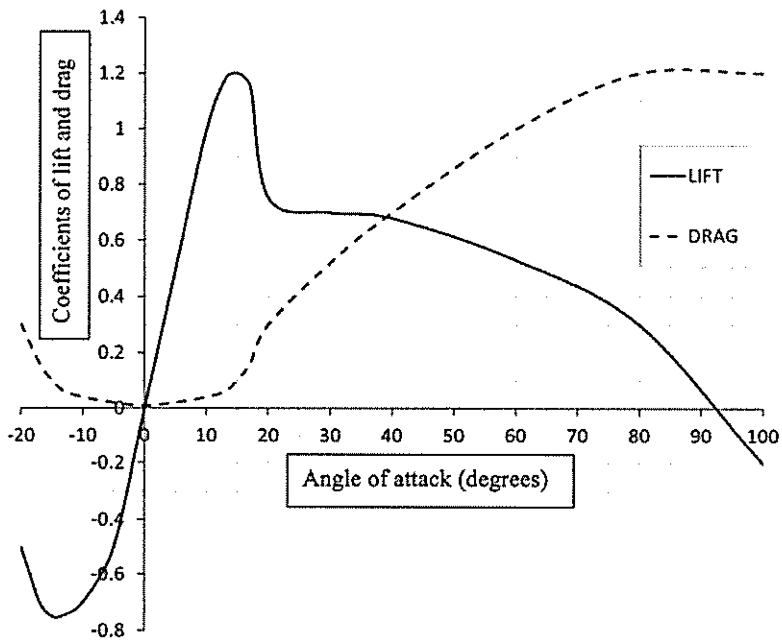

Fig. 5

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SECTION D: ***Aerothermal Engineering***

Answer not more than two questions from this section.

9 A simple turbojet engine operates with an inlet stagnation temperature of 288 K and a stagnation pressure of 0.98 bar. The design stagnation pressure ratio is 24 and both the compressor and turbine have isentropic efficiencies of 90%.

(a) If the compressor stage loading coefficient $\Delta h_0/U^2$ is not to exceed 0.45 and the mean blade speed at design is 300 m s^{-1} calculate the minimum number of compressor stages required. [5]

(b) The inlet blade height of the first compressor rotor is 20 cm and the mass flow rate is 60 kg s^{-1}. If the flow is axial at inlet with a Mach number of 0.58, find the compressor mean radius. [5]

(c) The turbine entry temperature is 1500 K. Calculate the stagnation temperature and the stagnation pressure at exit from the turbine. If the ambient pressure is equal to the engine inlet stagnation pressure, determine the engine jet velocity. [7]

(d) Explain why modern civil passenger aircraft use bypass engines rather than turbojets. Using a sketch, describe the main features of a high bypass ratio engine and state the factors that limit the upper value of the bypass ratio. [8]

Neglect any losses in the ductwork and the propelling nozzle. Assume that the working fluid is air throughout with $\gamma = 1.4$, $R = 287$ J kg^{-1}K^{-1} and $c_p = 1005$ J kg^{-1}K^{-1}.

10 (a) In the ICAO standard atmosphere, for altitudes above the tropopause, in the stratosphere, the ambient temperature is uniform and equal to T_T. Show that above the tropopause the pressure p at an altitude z is given by the expression

$$p = p_T \exp\{-\frac{g}{RT_T}(z - z_T)\}$$

where p_T is the ambient pressure at the tropopause, z_T is the altitude of the tropopause and g is the acceleration due to gravity. [5]

(b) Show that the mass m of an aircraft during cruise is given by

$$m = m_1 \exp\{-\frac{g\, SFC}{V\, L/D}\, s\}$$

where s is the cruise distance and m_1 is the aircraft mass at the start of cruise. Assume that during cruise the engine specific fuel consumption SFC, the flight speed V and the aircraft lift-to-drag ratio L/D are all constant. [5]

(c) Using the results from parts (a) and (b), show that for an aircraft to cruise at a constant lift coefficient above the tropopause, the cruise altitude varies linearly with cruise distance. [6]

(d) An aircraft with a total wing area of 270 m^2 starts cruise at altitude z_T. The aircraft cruises at a constant lift coefficient $C_L = 0.5$ with $SFC = 0.016$ kg s^{-1} kN^{-1} and $L/D = 20$. If the aircraft mass at the start of cruise is 150 tonnes and the aircraft cruises a distance of 6000 km, find the cruise Mach number, the mass of fuel burned during cruise and the altitude at the end of cruise. [9]

Take $\gamma = 1.4$ and $R = 287$ J kg^{-1} K^{-1} for air. Use the following data for conditions at the tropopause: $z_T = 11000$ m, $T_T = 216.65$ K, $p_T = 0.226$ bar.

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11 (a) An aircraft powered by turbofan engines cruises at a Mach number of 0.85 at an altitude where the ambient pressure is 23.8 kPa and the ambient temperature is 219 K. Find the stagnation pressure and stagnation temperature at engine inlet and the aircraft flight speed. [4]

(b) During cruise the engines that power the aircraft have a thermal efficiency of 48% and a propulsive efficiency of 84%. If the aircraft is powered by 4 engines and requires a total net thrust of 180 kN, determine the mass flow rate of air through each engine and the engine gross thrust. Also calculate the engine specific fuel consumption, assuming that the fuel has a lower calorific value of 43 MJ kg^{-1}. [8]

(c) The same turbofan engine is tested at sea level on a static test bed, where the ambient pressure is 102 kPa and the ambient temperature is 290 K. It is run at the same non-dimensional operating point as the cruise condition, for which both the bypass and core propelling nozzles are choked. If the total nozzle area A_N is 3.8 m^2 , determine the engine air mass flow rate, the engine gross thrust and the engine specific fuel consumption during the test. [9]

(d) State how the non-dimensional operating point of an engine is different at top-of-climb compared to cruise. Why are both the top-of-climb and cruise conditions important in the design of a turbofan engine? [4]

Take $\gamma = 1.4$, $R = 287$ J $\text{kg}^{-1}\text{K}^{-1}$ and $c_p = 1005$ J kg^{-1} K^{-1} for air. Note that the dimensionless mass flow rate, thrust and fuel flow are given by

$$\frac{\dot{m}\sqrt{c_p T_{02}}}{A_N p_{02}}, \quad \frac{F_G + p_a A_N}{A_N p_{02}} \text{ and } \frac{\dot{m}_f LCV}{A_N p_{02}\sqrt{c_p T_{02}}}$$

where $\dot{m}$ is the engine air mass flow rate, T_{02} is the engine inlet stagnation temperature, p_{02} is the engine inlet stagnation pressure, F_G is the gross thrust, p_a is the ambient pressure, $\dot{m}_f$ is the mass flow rate of fuel and LCV is the fuel lower calorific value.

SECTION E: *Electrical Engineering*

*Answer not more than **two** questions from this section.*
Note Data Sheet at end of the paper.

12 (a) The manufacture of MOSFETs in CMOS technology for operation in gigahertz frequencies requires several lithographic steps. Discuss the relative merits of UV photolithography and electron beam lithographic processes for achieving nanometre scale MOSFET gate lengths. [5]

(b) If a UV photolithographic source centred on 200 nm is used, determine the value of Numerical Aperture required for the optical system for a minimum feature size of 100 nm. [5]

(c) If an electron beam system with energy of 40 keV is utilised, estimate the resolution limit and explain why, in practice, this cannot be achieved. [5]

(d) A 40 nm layer of SiO_2 is required to be grown as a gate dielectric on a silicon wafer of thickness 475 μm. If a dry oxidation process is used, estimate the thickness left after the oxide is grown and also the thickness of the oxide plus silicon. [8]

(e) Suggest two dielectric materials, which could augment SiO_2 as the gate dielectric in MOSFETs as dimensions continue to shrink. [2]

13 (a) Explain, with aid of diagrams, the charge distribution in the channel of a depletion mode FET in the on and off states and how the gate voltage turns the FET off. [7]

(b) A gate-all-around FET consists of a rod-like Si nanowire of radius R doped to a dopant density of N. By Gauss's law or otherwise, derive an expression for the electric field at the surface in the off state, and thus the gate turn-off voltage. [10]

(c) Calculate the gate turn-off voltage for $N = 10^{25}$ m^{-3} and $R = 10$ nm. The relative dielectric constant of Si is 12. Comment on the value of this voltage. [8]

14 (a) Describe the typical velocity versus field curve for electrons in a semiconductor. Explain what is the scattering limited velocity and how it arises. [6]

(b) It is desired to make a thin film transistor (TFT) driver out of the layered semiconductor WSe_2 for an OLED device in a medical application whose response time should be no more than 10 μs. Derive the gate length that will give the transit time of 10 μs if the mobility of WSe_2 is 18 $cm^2\ V^{-1}\ s^{-1}$ and the source-drain voltage is 0.2 V. [6]

(c) Derive the carrier velocity and source-drain field at this gate length. [5]

(d) A layer of WSe_2 has a thickness of 0.336 nm. The electrodes induce a uniform mobile charge carrier density of $10^{26}\ m^{-3}$ in the channel. How many layers thick should the channel be in order to give a source-drain current of 1 mA, if the gate width to gate length ratio cannot be larger than 50? [8]

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SECTION F: *Information Engineering*

*Answer not more than **two** questions from this section.*

15 (a) We wish to find an image value at position (p,q) via *bilinear interpolation* using the 4 pixels surrounding the point (p,q). Suppose these surrounding pixels are at (a,c), (b,c), (a,d) and (b,d), with image values of x_{ac}, x_{bc}, x_{ad}, x_{bd} respectively, as shown in Fig. 6.

(i) This bilinear interpolation results in an image value at (p,q) given by an expression of the form

$$x_{pq} = \alpha x_{ac} + \beta x_{bc} + \gamma x_{ad} + \delta x_{bd}$$

By using 1D linear interpolation on the rows then on the columns (or vice versa), derive the values of the constants $\alpha, \beta, \gamma, \delta$. [8]

(ii) Another interpolation scheme for images is *bicubic interpolation*. Explain *qualitatively* how we perform bicubic interpolation on an image and discuss the advantages and disadvantages over bilinear interpolation. [5]

(b) Gaussian filters are frequently used to low-pass filter images. The Gaussian filter given below

$$g(u) = \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-u^2/2}$$

has a frequency response, $G(\omega)$, given by $G(\omega) = \mathrm{e}^{-\omega^2/2}$

(i) Suppose we filter an image with a 2D filter $g_{rc}(x,y)$, given by

$$g_{rc}(x,y) = \frac{1}{b}\, g\left(\frac{x}{b}\right) \frac{1}{b}\, g\left(\frac{y}{b}\right)$$

where b is a scale factor. Explain why this 2D filter is *isotropic*, i.e. has the same effect on the image in all directions. [6]

(ii) Describe the effect of the scale factor b on the frequency response of the filter and on the filtered image. [6]

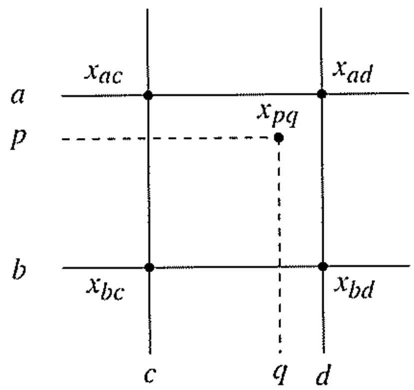

Fig. 6

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16 (a) A grey scale image, $I(x,y)$, is smoothed before image gradients are computed as part of the feature detection and matching process.

(i) What smoothing filter is used in practice? Give an expression for computing the intensity of a smoothed pixel, $S(x,y)$, with two discrete 1D convolutions. [5]

(ii) How are the 2D image gradients computed. Show how differentiation can also be performed by two discrete 1D convolutions and identify the filter coefficients. [4]

(iii) Show how different resolutions of the image can be represented efficiently in an *image pyramid*. Your answer should include details of the implementation of smoothing within an octave and sub-sampling of the images between octaves. [6]

(b) Consider an algorithm to detect and match interest points (features of interest) in a 2D image.

(i) Show how image features such as *blob-like* shapes can be localised in position and scale by *band-pass* filtering. [4]

(ii) How are these features localised efficiently using the image pyramid? [4]

(iii) How is the dominant orientation of each feature determined? [2]

17 (a) A mobile phone application is used to match an image of a planar target, such as a book cover, by searching images of known planar objects in its database. A finite number of interest points is extracted automatically from each image (along with the appropriate scale and orientation) and then each feature is characterised by its Shift Invariant Feature Transform descriptor.

(i) Describe the main steps in computing this descriptor from the image. [5]

(ii) What properties of the neighbourhood of each feature is this descriptor encoding? [2]

(iii) Give details of an algorithm used to find correspondences from the mobile phone image and the target's reference image in the database. [4]

(iv) How is the correct target determined? [2]

(b) A *convolutional neural network* is to be used to determine the categories of objects in the image.

(i) Describe the architecture of a typical *convolutional neural network* used in object recognition. Include details of the number of layers, the sizes of the filter kernels and the role of any non-linear elements in the network. [5]

(ii) Describe how the parameters of the network are estimated by *supervised learning* with labelled images of known object categories. [3]

(iii) Show how an arbitrary image can be represented by a vector of fixed length by the network. [2]

(iv) How is the category of the object in the image assigned for unseen test data? [2]

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SECTION G: ***Bioengineering***

Answer not more than **two** *questions from this section.*

18 (a) Explain the *pulse-echo* technique used to image the eye, both with optical and ultrasound methods. Discuss the main differences between ultrasound imaging and optical imaging. [7]

(b) Discuss the attenuation properties of opthalmic ultrasound systems, indicating how such properties are affected by the frequency of the ultrasound signal. [3]

(c) When imaging the eye with a variety of modalities, a 2D cross-sectional image is created, with the x-axis representing lateral distance across the eye and the y-axis representing depth into the eye. *Scanning Laser Opthalmoscopy* (SLO) is one such modality.

(i) Briefly describe the technique of SLO and discuss its advantages over a Fundus Camera. [6]

(ii) For SLO, explain what determines the resolution in both x and y directions, giving typical values for these resolutions. [3]

(iii) Explain how SLO uses *confocal* optics to reduce the depth range for a given focus. [6]

19 (a) The cornea of the eye is largely comprised of the natural material collagen. Describe the structure and self-assembly of collagen. Explain how the collagen in cornea and sclera differ. [6]

(b) Describe two different ways of mechanically testing the cornea and explain the advantages and disadvantages of each. [5]

(c) Describe the structure of the crystalline lens. Explain the process of lens accommodation and how it changes with ageing. [6]

(d) Describe the transport mechanism in poroelasticity. If a cornea has a thickness h of 400 μm, an elastic modulus E of 500 kPa, and an intrinsic permeability k of 2×10^{-16} m^2, what is the time constant for poroelastic transport through the cornea, assuming the viscosity of water η is 1 mPa s? If the cornea thickness decreases to 200 μm, what effect does this have on the transport behaviour? If the permeability stayed constant, how would the modulus have to change to restore the original transport response in this thinner tissue? [8]

20 (a) Write short notes on:

(i) rods and cones;

(ii) the retinotopic arrangement of neurons in the primary visual cortex;

(iii) orientation maps in the primary visual cortex. [6]

(b) Sketch and explain the receptive field properties of a typical retinal ganglion cell. Label each relevant component of your sketch clearly. [3]

(c) In what form does the brain receive visual information from a retinal ganglion cell? Explain whether this information is mostly about mean luminance or luminance contrast, and why this information is computationally useful? [3]

(d) In the context of vision in vertebrates, explain what the "physiological blind spot" is, and the mechanism underlying it. Describe an experiment with which you could demonstrate the existence of the blind spot in your left eye. [3]

(e) Write short notes on:

(i) everse versus inverse retinas;

(ii) spherical aberration. [4]

(f) What is the distribution, $P(n)$, of (integer) spike counts, $n \geq 0$, in a receptor that maximises its response entropy with the constraint that its average spike count is $\bar{n}$? [6]

SECTION H: *Manufacturing and Management*

Answer not more than two questions from this section.

21 (a) What is the maximum length of time for which a U.K. patent can be granted? [1]

(b) Describe the three tests that an invention must satisfy in order for it to be considered patentable. [4]

(c) A team of students is planning to commercialise a new tracking technology that helps managers locate key items of equipment in hospitals, factories and airports. The technology combines small, low-cost wireless tags with software based upon a novel algorithm. The technology is going to be marketed using the name '*AssetTracker*'.

(i) Discuss the Intellectual Property (IP) issues that the team will need to consider when they attempt to commercialise this technology.

(ii) Discuss the possible business models that could be used to commercialise this technology.

[20]

22 (a) Explain what is meant by:

(i) functional decomposition; and

(ii) product specification. [6]

(b) Describe the different types of prototyping and the role each type can play in the design process. [10]

(c) Discuss the links between the design of a product and the design of the manufacturing process for making that product. [9]

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23 (a) Describe what is meant by:

(i) technology push; and

(ii) market pull.

Give four examples each for (i) and (ii). [6]

(b) Compare, using examples, the relative benefits of using business models based on:

(i) licensing of IP;

(ii) manufacturing and selling a product; and

(iii) selling a service. [9]

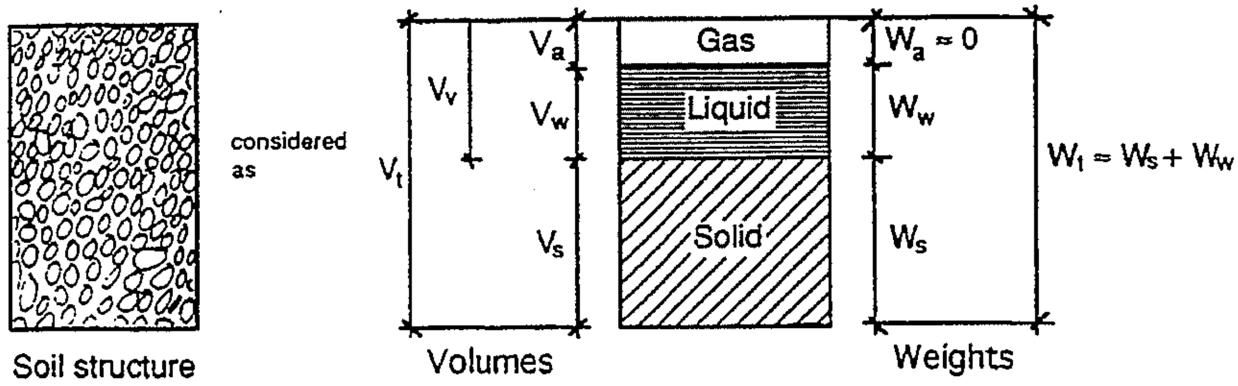
(c) Compare the challenges of managing innovation in a small start-up company with those of managing innovation in a large, long-established firm. [10]

END OF PAPER

Engineering Tripos Part IB Civil Engineering Elective

Data sheet – Soil Mechanics

General definitions



Specific gravity of solid	G_s
Voids ratio	$e = V_v / V_s$
Specific volume	$v = V_t / V_s = 1 + e$
Water content	$w = (W_w / W_s)$
Degree of saturation	$S_r = V_w / V_v = (w\,G_s / e)$
Unit weight of water	$\gamma_w = 9.81$ kN/m^3(although we assume 10 kN/m^3)
Unit weight of soil	$\gamma = W_t / V_t = \left(\dfrac{G_s + S_r e}{1+e}\right)\gamma_w$
Buoyant (effective or submerged) unit weight	$\gamma' = \gamma - \gamma_w = \left(\dfrac{G_s - 1}{1+e}\right)\gamma_w$ (soil saturated)
Unit weight of dry soil	$\gamma_d = W_s / V_t = \left(\dfrac{G_s}{1+e}\right)\gamma_w$
Relative density	$I_d = \dfrac{(e_{max} - e)}{(e_{max} - e_{min})}$

where e_{max} is the maximum voids ratio achievable in the quick tilt test (for sands), and
e_{min} is the minimum voids ratio achievable by vibratory compaction (for sands).

Classification of particle sizes

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

D equivalent diameter of soil particle

D_{10}, D_{60} etc particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.

Stress components

Principle of effective stress (saturated soil):

total stress	σ	=	effective stress σ' +	pore water pressure	u
	τ	=	τ' +		0

and

σ_v = vertical stress

σ_h = horizontal stresss

τ = shear stress

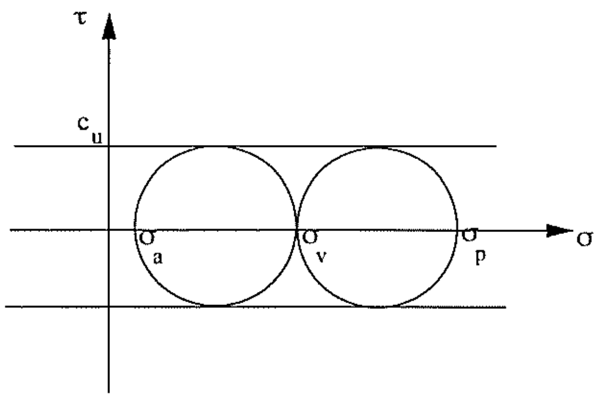
Strength of clays (undrained behaviour only)

Under constant volume (undrained) conditions only, the strength of clays can be characterised by the *undrained shear strength* c_u which is mobilized when the shear stress $\tau = c_u$. This conforms to Tresca's criterion, and the active and passive total horizontal stresses, σ_a and σ_p respectively, are given by

$$\sigma_a = \sigma_v - 2\,c_u$$

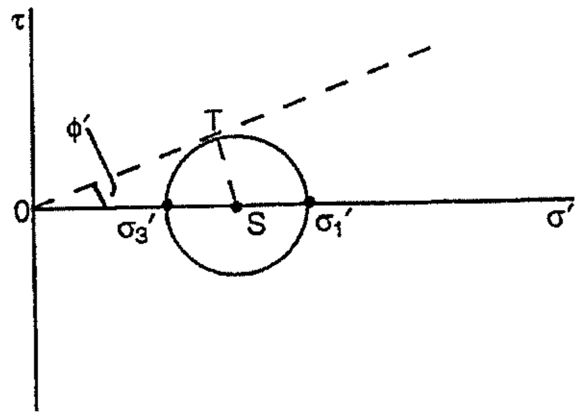
$$\sigma_p = \sigma_v + 2\,c_u$$

where σ_v is the total vertical stress.



Strength of sands

Mobilised angle of shearing ϕ'
where $\tau = \sigma' \tan \phi'$



$$\sin\phi' = TS/OS$$

$$= \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

$$\therefore \phi' = \sin^{-1}\left[\frac{\left(\sigma_1'/\sigma_3'\right) - 1}{\left(\sigma_1'/\sigma_3'\right) + 1}\right]$$

Earth pressure coefficient K: $\sigma_h' = K\sigma_v'$

Active pressure: $\sigma_v' > \sigma_h'$ $\therefore$ $\sigma_1' = \sigma_v'$

$\sigma_3' = \sigma_h'$

$K_a = (1 - \sin\phi'/1 + \sin\phi')$

Passive pressure: $\sigma_h' > \sigma_v'$ $\therefore$ $\sigma_1' = \sigma_h'$

[We assume principal stresses $\sigma_3' = \sigma_v'$

are horizontal and vertical] $K_p = (1 + \sin\phi')/(1 - \sin\phi') = \frac{1}{K_a}$

Angle of shearing resistance:

at peak strength ϕ'_{max} at $\left(\sigma_1'/\sigma_3'\right)_{max}$

at critical state ϕ'_{crit} after large strains.

Sand strength data: friction hypothesis

In any shear test on soil, failure occurs when $\phi' = \phi'_{max}$ and

$$\phi'_{max} = \phi'_{crit} + \phi'_{dilatancy}$$

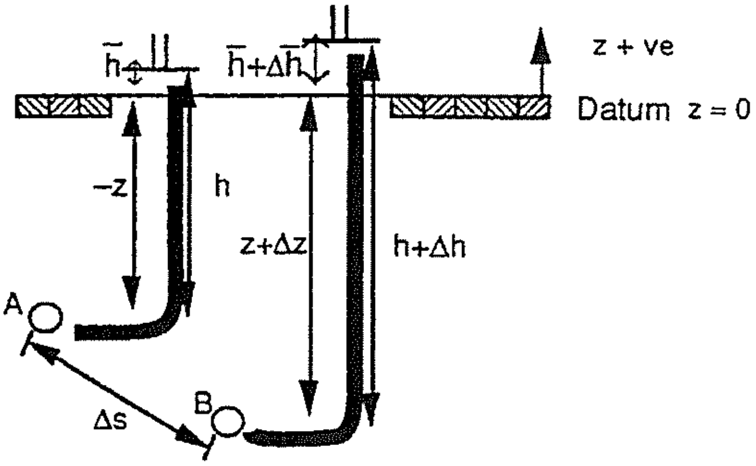
where ϕ'_{crit} is the ultimate angle of shearing resistance of a random aggregate which deforms at constant volume, so the dilatancy, which indicates an increase in volume during shearing, approaches zero $(\phi'_{dilatancy} -> 0)$ as $\phi'_{max} -> \phi'_{crit}$.

ϕ'_{crit} is a function of the mineralogy, size, shape and distribution of particles. For a particular soil it is almost independent of initial conditions. Typical values (±2°):

	ϕ'_{crit}	ϕ'_{max}	
feldspar	40°		
quartz	33°	53°	($I_d = 1$, and mean effective stress OS < 150 kPa)
mica	25°		

Seepage

Excess pore water pressure



Head $h = u/\gamma_w$

Potential $\bar{h} = h + z$

Total pore water pressure head at A: $u = \gamma_w h = \gamma_w (\bar{h} - (-z))$

B: $u + \Delta u = \gamma_w (h + \Delta h) = \gamma_w (\bar{h} + z + \Delta\bar{h} + \Delta z)$

[Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta\bar{u} = \gamma_w (\bar{h} + \Delta\bar{h})$]

Hydraulic gradient A–B $i = -\dfrac{\Delta\bar{h}}{\Delta s} = -\dfrac{\Delta\bar{u}}{\gamma_w \Delta s}$

Darcy's law $v = ki$

v = average or superficial seepage velocity

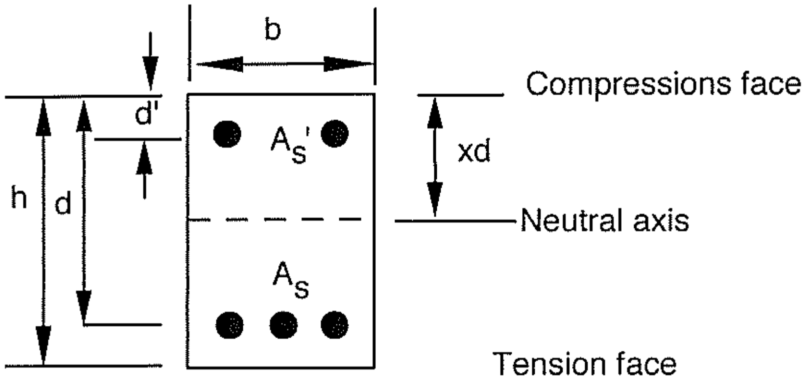
k = coefficient of permeability

Typical permeabilities

$D_{10} > 10\text{mm}$	:	non-laminar flow
$10\text{ mm} > D_{10} > 1\mu\text{m}$	:	$k \cong 0.01(D_{10} \text{in mm})^2$ m/s
clays	:	$k \cong 10^{-9}$ to 10^{-11} m/s

Design of reinforced concrete

Data sheet for use in Part IB Civil Engineering Elective Course.



Design Stresses

Cube strength for concrete f_{cu}. At failure in bending, stress in concrete = $0.4f_{cu}$ over whole area of concrete in compression.

Tensile yield stress of steel f_y. At failure in bending, stress in bars in tension = $0.87f_y$, stress in bars in compression = $0.75f_y$.

Design Equations

Moment capacity of singly reinforced beam

$$M \le 0.15 f_{cu} bd^2$$
$$M = 0.87 f_y A_s d(1 - x/2)$$
$$x = 2.175(f_y / f_{cu}).(A_s / bd) \quad (\le 0.5 \text{ to avoid over reinforcement})$$

Moment capacity of doubly reinforced beam

$$M = 0.15 f_{cu} bd^2 + 0.75 f_y A'_s (d - d')$$
$$0.87 f_y A_s = 0.75 f_y A'_s + 0.2 f_{cu} bd$$

Shear capacity of all beams

Total shear capacity $V = (v_c + v_s)bd$

Where, $v_c = 0.68(100A_s/bd)^{0.33}.(400/d)^{0.25}$ (N/mm^2)

and $v_s = 0.87f_y A_{sq}/(bs)$

in which s = shear link spacing, A_{sq} is total area of all shear bars in a link and A_s is the total area of effective longitudinal *tension* steel at the section.

Standard bar sizes

Diameter (mm)	6	8	10	12	16	20	25	32	40	50
Area (mm^2)	28	50	78	113	201	314	491	804	1256	1963

Available steel types

Deformed high yield steel $f_y = 460$ N/mm^2

Plain mild steel $f_y = 250$ N/mm^2

Lap and anchorage lengths 40 bar diameters

Density of reinforced concrete: 24 kN/m^3

Reinforcement areas per metre width

	Spacing of bars (mm)									
	75	100	125	150	175	200	225	250	275	300
Bar Dia. (mm)										
6	377	283	226	189	162	142	126	113	103	94.3
8	671	503	402	335	287	252	224	201	183	168
10	1050	785	628	523	449	393	349	314	285	262
12	1510	1130	905	754	646	566	503	452	411	377
16	2680	2010	1610	1340	1150	1010	894	804	731	670
20	4190	3140	2510	2090	1800	1570	1400	1260	1140	1050
25	6550	4910	3930	3270	2810	2450	2180	1960	1790	1640
32	10700	8040	6430	5360	4600	4020	3570	3220	2920	2680
40	16800	12600	10100	8380	7180	6280	5580	5030	4570	4190
50	26200	19600	15700	13100	11200	9820	8730	7850	7140	6540
Areas calculated to 3 significant figures according to B.S.I recommendations										

Part IB Data Sheet: Electrical Engineering Elective
Transistor Design Summary Sheet

Gauss's Theorem

$\varepsilon_0\varepsilon_r E_1 - \varepsilon_0\varepsilon_r E_2 =$ charge per unit area enclosed between upper surface 1 and lower surface 2.

FET Design Summary

- $\tau_t \rightarrow$ switching time as 1st approx. (scattering limited transit time).
- $\tau_{eff} = \tau_t + R_{load}C_{eff(output)} \rightarrow$ switching time as 2nd approx.
- $L = v_s\tau_t$ (source-drain spacing).
- $I_{sat} = eNv_s W d_s = eN W L d_s/\tau_t$
- Aspect ratio W/L (technology?).
- $(1/2)eN(d_s)^2/\varepsilon_0\varepsilon_r =$ (Max Gate Voltage)
- $E'_{peak} = eNd_s/\varepsilon_0\varepsilon_r < E_{breakdown}$
- Minimum Drain Source Voltage $\sim E_s L$ (E_s is the field required to reach limiting velocities).

Mutual Conductance

$$g_{mo} \sim I_{sat}/V_{gate(max)}$$

Mutual conductance reduces with frequency as $g_m(\omega) \approx g_{mo}/(1 + j\omega\tau_t)$;

$$v_{out} = g_m(\omega)\, R\big(1 + j\omega RC_{eff(out)}\big) \approx g_{mo}R/[1 + j\omega(\tau_t + RC_{eff(out)})]$$
$$= g_{mo}R/[1 + j\omega\tau_{eff}]$$

Capacitances for FET

Parallel Plate Capacitance: $\varepsilon_0\varepsilon_r$ Area/spacing

Used for rough estimates of parasitic capacitance.

Effective Capacitances for FET

$$C_{eff(out)} \rightarrow C_{gate/drain} + C_{drain/source} + C_{load};$$
$$C_{eff(in)} \rightarrow MC_{gate/drain} + C_{gate/source\,(proximity)} + C_{gate/source\,(electronic)};$$
$$C_{electronic} = g_{mo}\tau_t \quad ; \quad M = (1 + g_{mo}R_{load}).$$

Time Constants for FET

$\mu = e\tau/m^*$ relates mean free time τ and mobility.

Transit time τ_t over distance L and scattering limited velocity v_s are related by $\tau_t = L/v_s$.

$v_{out} \approx g_{mo}R/[1 + j\omega\tau_{eff}] = g_{mo}R/[1 + j\omega/(2\pi f_t)]$

$1/(2\pi f_\tau) = \tau_t + RC_{eff(out)} = \tau_{eff}$ The transition frequency is f_t.

10% to 90% rise time is $T = 2.2\,\tau_{eff} = (2.2/2\pi)(1/f_t) = 0.35/f_t$.

Electrical Engineering Elective:
Tunnel Barrier Design Summary Sheet

Schrodinger's Equation

Complex Wave $\Psi = A\ \exp(-j2\pi f\ t + j2\pi x/\lambda) = A \exp(-j\omega t)\exp(jkx)$;
<momentum> $\Psi = p\Psi = (h/\lambda)\Psi = -j(h/2\pi)\partial\Psi/\partial x$;

<Total energy> $\Psi = E\Psi = hf\Psi = j(h/2\pi)\partial\Psi/\partial t$
$(h/2\pi) \rightarrow \hbar$; $h = 6.625 \times 10^{-34}$ J/s.

Schrodinger's equation:-

$$E\Psi = (1/2m)[-j\hbar\partial/\partial x]^2\Psi + e\phi\Psi$$

Tunneling (Rectangular barrier $\varepsilon\phi$)

Propagating waves outside barrier with incident kinetic energy $U_{incident} = (\hbar k_i)^2/2m^*$

Evanescent waves inside barrier: $-(\hbar k)^2/2m^* = (\hbar k_i)^2/2m^* - e\phi$

Technology Design Summary

Diffusion

Constant Surface Concentration:

$$C(x,t) = C_s\ \text{erfc}\left[\frac{x}{2(Dt)^{1/2}}\right]$$

Constant Total Dopant:

$$C(x,t) = \frac{S}{(\pi Dt)^{1/2}} \exp\left[\frac{-x^2}{4Dt}\right]$$

W I Milne
May 2011

Engineering Tripos Part IB 2016
Paper 8 Selected Topics

Answers

SECTION B – Civil

3. (a)(i) 44%, 17.4 kNm^{-3}
(a)(ii) 174 kNm^{-2}, 85.8 kNm^{-2}
(a)(iii) 274 kNm^{-2}
(a)(iv) 299 kNm^{-2}
(b)(ii) 15 m

4.
(a)(ii) 2.25, 1.33
(a)(iii) 65.6 kNm^{-1}

5 (b) 765 mm, 9,329 mm^2/m, 15,230 mm^2/m

SECTION C – Mechanics, Materials and Design

7. (a) (i) 4, 0.2, 84.8 m, (ii) 39.3 rad/s, 35
(b) (i) -30.5 kNm, -0.0275, 107 A, (ii) -7.63 kNm, -0.00688, 26.4 A, (c) (i) 2.4%

8 (b) 4440 Nm, (c) 0.012 rad/s^2, (d) 260 s

SECTION D - Aerothermal

9. (a) 12 stages
(b) 0.248 m
(c) 1027 K, 5.189 bar, 884.2 m/s

10. (d) 0.83, 26.24 tonne, 12.22 km

11. (a) 38.17 kPa, 250.6 K, 252 m/s
(b) 468.5 kg/s, 163.1 kN, 14.54 g $kN^{-1}s^{-1}$
(c) 1164 kg/s, 289.9 kN, 6.49 g $kN^{-1}s^{-1}$

SECTION E – Electrical

12 (b) Resolution = 0.5(λ/NA). For a 200 nm source, 100 ~ 0.5x200/NA and NA = 1.
12 (c)

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.625\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times1.6\times10^{-19}\times40\times10^{13}}}$$

$$\lambda = \sim 6\times10^{-12}\,m$$

12(d) New Si thickness = (475-0.018) microns = 474.982 microns
New total thickness = 475+(0.55x0.040) = 475.022 microns

13 (b) $2.\varepsilon.E = N.e.r$ and $V = -N.e.r^2/(4.\varepsilon)$
13 (c) $V = 10^{25} \times 1.6.10^{-19} \times (10^{-8})^2/(4 \times 12 \times 8.8.10^{-12}) = 0.379$ V.

14 (b) $\tau = L^2/\mu.V$ or $L^2 = t.\mu.V$, Gate length $L = 6\times10^{-5}$ m.
14 (c) $E = 0.2/\ 6\times10^{-5} = 3.33\times10^3$ V/m, $v = E.\mu = 6$ m/s.
14 (d) Number of layers = 3.47 / 0.336 = 10.33 →11.