1)

(a) $(T+a)v = b(T_0 - T)$

$$v = \frac{b(T_0 - T)}{T+a}$$

Power $vT = \dfrac{bT(T_0 - T)}{T+a}$

max power $\dfrac{d}{dT}(vT) = 0$, $P = vT$

$$\frac{dP}{dT} = 0 = \left(\frac{1}{a} + \frac{1}{T}\right) = \left(\frac{T_0}{T^2} - \frac{1}{T}\right)$$

$$T = \frac{-2a \pm \sqrt{4a^2 + 4aT_0}}{2}$$

$$v_{opt} = \frac{b\left[1 + \frac{a}{T_0} - \sqrt{\frac{a^2}{T_0^2} - \frac{a_0}{T_0}}\right]}{\sqrt{\frac{a^2}{T_0^2} + \frac{a}{T_0}}}$$

A cyclist choses a gear so that muscle velocity $= v_{opt}$ so as to maximise power o/p.

(b) Thermal activation of molecules leads to progressive loss of correlation along the length of the molecule. The persistence length ξ is the length over which correlation is lost by thermal activation

$$kT \sim \frac{1}{2} EI \phi^2 / \xi$$

where ϕ is end-to-end rotation

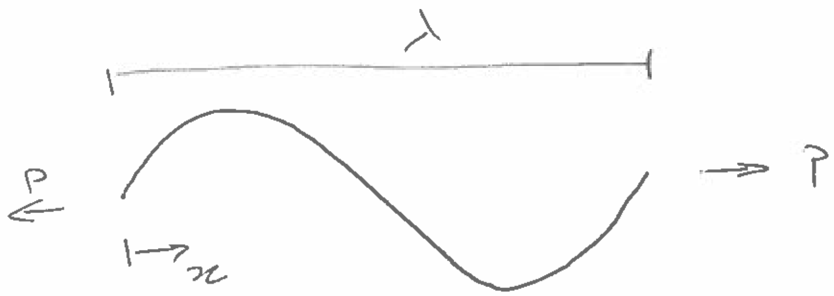
$$\Rightarrow \xi = \frac{EI}{2kT} \quad \text{taking } \phi \sim 1.$$

Stiff molecules like microtubules have long persistence lengths & remain straight over the length of the cell while intermediate filaments with small ξ coil randomly

(c) The cellulose cell wall in plants gives it structural stiffness. Animal cells do not have this & rely on the cytoskeleton for structural support.

2

(a)



waviness $\quad w(x) = a \sin\frac{2\pi x}{\lambda}$

$$M(x) = Pw = EI\frac{d^2u}{dx^2}$$

$$\frac{d^2u}{dx^2} = \frac{Pa}{EI}\sin\frac{2\pi x}{\lambda}$$

$$u = -\left(\frac{\lambda}{2\pi}\right)^2\frac{Pa}{EI}\sin\frac{2\pi x}{\lambda}$$

assume $\quad P \ll \frac{EI\lambda^2}{4\pi^2} \Rightarrow |u| \ll |w|$

$$e = -\frac{1}{\lambda}\int_0^\lambda \left\{\left[1+\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial x}\right)^2\right]^{\frac{1}{2}} - \left[1+\left(\frac{\partial w}{\partial x}\right)^2\right]^{\frac{1}{2}}\right\}dx$$

$$\frac{\partial u}{\partial x} = \frac{-\lambda}{2\pi}\frac{Pa}{EI}\cos\frac{2\pi x}{\lambda}$$

$$\frac{\partial w}{\partial x} = \frac{2\pi a}{\lambda}\cos\frac{2\pi x}{\lambda}$$

$$\left[1+\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial x}\right)^2\right]^{\frac{1}{2}} \sim 1+\frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2+2\frac{\partial u}{\partial x}\frac{\partial w}{\partial x}+\left(\frac{\partial w}{\partial x}\right)^2\right]$$

$$\left[1+\left(\frac{\partial w}{\partial x}\right)^2\right]^{\frac{1}{2}} \sim 1+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2$$

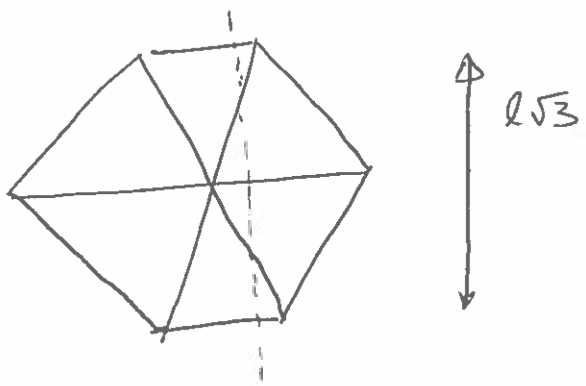
$$e_\lambda \approx -\int_0^\lambda dx \left(\frac{\partial u}{\partial x}\frac{\partial w}{\partial x}\right)$$

$$= \frac{\lambda}{2}\frac{Pa^2}{EI}$$

$$e = \frac{Pa^2}{2EI} \quad ; \quad EI = E_s \frac{\pi}{4}\left(\frac{d}{2}\right)^4$$

$$\Rightarrow e = \frac{32}{\pi}\frac{a^2 P}{E_s d^4}$$

(b) By symmetry $T_B = T_F = T_E = T_C$ & $T_A = T_D$



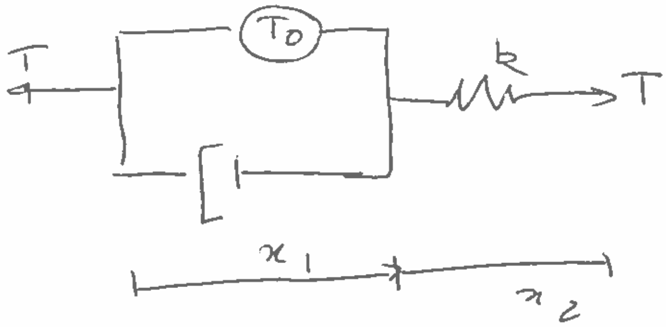
& $T_B = T_F = T_E = T_C = 0$

$$\Rightarrow T_A = T_D = \sqrt{3}\,\ell d\,\Sigma_{11}$$

(c) $E_1 = \dfrac{\Sigma_{11}}{e} = \dfrac{\pi}{32\sqrt{3}}\dfrac{E_s d^3}{a^2 \ell}$

(3)

(a)



$$k\,\Delta x_2 = T \quad \& \quad B\dot{x}_1 + T_0 = T$$

Since muscle under isometric tension $\dot{x}_1 = -\dot{x}_2 = \frac{-\dot{T}}{k}$

$$T + \frac{B}{k}\dot{T} = T_0$$

with $T(0) = 0$

$$\Rightarrow \quad T(t) = T_0\left[1 - e^{-\frac{kt}{B}}\right] \qquad 0 \leq t \leq C$$

(b) $$T(C) = T_0\left[1 - e^{-\frac{kC}{B}}\right]$$

$$\& \quad T + \frac{B}{k}\dot{T} = 0$$

$$\Rightarrow T(t) = \left[1 - e^{-\frac{kC}{B}}\right] e^{-\frac{k(t-C)}{B}} \qquad C \leq t \leq C + A$$

(c) Problem is linear $\Rightarrow$ use superposition

$\Rightarrow$ between $C+A$ & $2C+A$ the response due to 2nd input is

$$T_0\left[1 - e^{-\frac{k(t-C-A)}{B}}\right]$$

& $T(t) = T(c)\, e^{-\frac{k(t-c)}{B}} + T_0 \left[1 - e^{-k\frac{(t-c-A)}{B}} \right]$

(d) The superposition nature of solution suggests that it will be able to capture the difference between a twitch & tetanus.

(4)

(a) Myoglobin stores O_2 & releases it when the environmental O_2 is low. This mechanism gives the enhanced diffusion rate rather than the actual faster transport of O_2

(b) Large molecules such as glucose are transported across the cell membrane via a carrier mediated transport mechanism such as uniports, symports etc.

(c)

(i) In its E1 conformation the Na^+/K^+ ATPase has 3 Na binding sites

4 (c) (i) In its El conformation, the Na+/K+ ATPase has three high-affinity Na-binding sites and two low-affinity K- binding sites accessible to the cytosolic surface of the protein. The Km for binding of Na+ to these cytosolic sites is 0.6 mM, a value considerably lower than the intracellular Na concentration of approx. 12mM; as a result, Na+ ions normally will fully occupy these sites. Conversely, the affinity of the cytosolic K-binding sites is low enough that K+ ions, transported inward through the protein, dissociate from E1 into the cytosol despite the high intracellular K concentration. During the E1-> E2 transition, the three bound Na+ ions become accessible to the exoplasmic face, and simultaneously the affinity of the three Na-binding sites becomes reduced. The three Na+ ions, transported outward through the protein and now bound to the low-affinity Na+ sites exposed to the exoplasmic face, dissociate one at a time into the extracellular medium despite the high extracellular Na concentration. Transition to the E2 conformation also generates two high-affinity K+ sites accessible to the exoplasmic face. Because the Km for K+ binding to these sites (0.2 mM) is lower than the extracellular K+ concentration (4 mM), these sites will fill with K+ ions. Similarly, during the E2 -> El transition, the two bound K+ ions are transported inward and then released into the cytosol.

(ii) Overall per ATP molecule hydrolyzed the pump moves 3 Na^+ ions & 2 K^+ ions. Increasing the ATP concentration will increase the rate of the pump.

Q1 Muscle power + qualitative reasoning of biological fibres

15 attempts, Average mark 73%

A question that was well-attempted. Most students calculated muscle power correctly but gave poor explanation of persistence length and the cell walls of plant and animal cells

Q2 Modulus of a network of wavy struts

5 attempts, Average mark 60%

Generally poorly attempted and few attempts too. The students struggled to calculate the effective modulus of a wavy strut and even in calculating strut forces in the network by method of sections

Q3 Hill muscle model

11 attempts, Average mark 67%

Generally well attempted but they struggled on using superposition to calculate the effect of multiple stimuli.

Q4 Qualitative question on ion pumps and transport mechanisms

14 attempts, Average mark 70%

Generally well attempted although the explanation of ATP pumps was generally not adequate.