Q1

(a) lift off occurs between 50 kPa and 75 kPa.
Extrapolating backwards from the readings at 75 kPa and 100 kPa, lift off can be estimated to occur at $\sigma_c \cong 67$ kPa.
Ignoring any corrections for membrane/installation

$$\sigma_{ho} = 67 \text{ kPa}$$

at 3m depth, the pore pressure is:

$$u_0 = 2.5 \times 10 = 25 \text{ kPa}$$

Therefore:

$$\sigma'_{ho} = \sigma_{ho} - u_0 = 67 - 25 = 42 \text{ kPa}$$

$$\sigma_{vo} = 3 \times 18 = 54 \text{ kPa}$$

$$\sigma'_{vo} = \sigma_{vo} - u_0 = 54 - 25 = 29 \text{ kPa}$$

$$\therefore \quad K_0 = \frac{\sigma'_{ho}}{\sigma'_{vo}} = \frac{42}{29} = 1.45$$

[20%]

(b) $K_{0,nc} = 1 - \sin\varphi' = 1 - \sin 23° = 0.61$

$$OCR^{0.9} = \frac{K_0}{K_{0,nc}} = \frac{1.45}{0.61} = 2.38 \Rightarrow OCR = 2.6$$

$$OCR = \frac{\sigma'_{vmax}}{\sigma'_{vo}} \Rightarrow \sigma'_{vmax} = OCR\, \sigma'_{vo} = 2.6 \times 29 \cong 76 \text{ kPa}$$

[20%]

(c) Volume of pressuremeter

$$V = A \cdot h$$

where A, area of cross section, πr^2
h, length of pressuremeter, constant

$$\ln V = \ln A + \ln h$$

$$\frac{\delta V}{V} = \frac{\delta A}{A} + \frac{\delta h}{h} = \frac{\delta A}{A} \qquad \left(\frac{\delta h}{h} = 0\right)$$

$$A = \pi r^2$$

$$\frac{\delta A}{A} = \frac{2\delta r}{r}$$

δr, change of radius
r, current radius

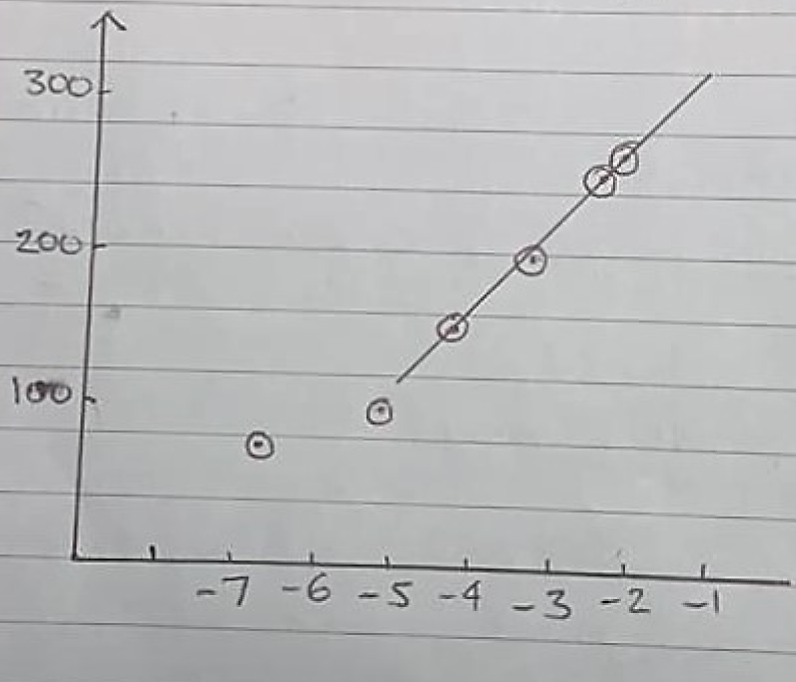
initial radius of cavity $r_0 = 41$ mm $(r = r_0 + \delta r)$

σ_c (kPa)	δr (mm)	$\delta A/A = 2\delta r/r$	$\ln \delta A/A$
0	0	0	–
25	0	0	–
50	0	0	–
75	0.02	0.0009	−6.9
100	0.08	0.0039	−5.5
150	0.31	0.0150	−4.2
200	0.90	0.043	−3.1
250	2.67	0.122	−2.1
270	4.10	0.182	−1.7

From Data Book:

$$\delta\sigma_c = s_u\left[1 + \ln\frac{G}{s_u} + \ln\frac{\delta A}{A}\right]$$

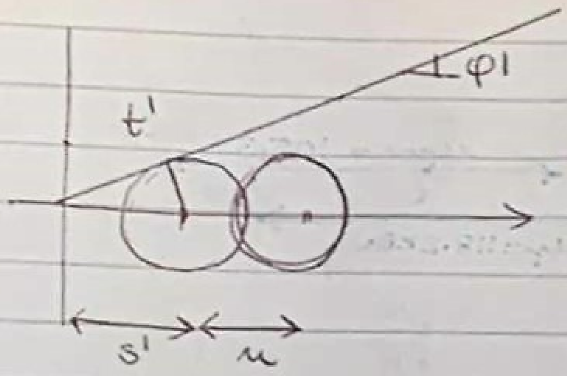
plot σ_c against $\ln\frac{\delta A}{A} \Rightarrow$ slope is s_u.



$$s_u = \frac{270-150}{4.2-1.7} = \frac{120}{2.5} = 48\,\text{kPa}$$

[30%]

(d) plane strain ($\varepsilon_z = \varepsilon_z = 0$), therefore
$\sigma_r = \sigma_1 \quad \sigma_\theta = \sigma_3$



$t' = s_u$
$t' = s' \sin\varphi'$
$s_u = s' \sin\varphi'$

$$s' = \frac{s_u}{\sin\varphi'} = \frac{48}{\sin 23} \simeq 123\,kPa$$

$\sigma_r = \sigma_c = 270\ kPa$
$\sigma_\theta = \sigma_r - 2s_u = 270 - 2\times 48 = 174\ kPa$

$$s = \frac{\sigma_r + \sigma_\theta}{2} = \frac{270+174}{2} = 222\ kPa$$

$u = s - s' = 222 - 123 = 99\ kPa$ [30%]

Q2 (a) before embankment construction, stress @A.

$\sigma_v = 18\times 8 = 144\ kPa$
$u = 10\times 7 = 70\ kPa$
$\sigma'_v = \sigma_v - u = 144 - 70 = 74\ kPa$
$\sigma'_h = K_0\sigma'_v = \sigma'_v = 74\ kPa$
$\sigma_h = \sigma'_h + u = 74 + 70 = 144\ kPa$

$t' = t = \frac{1}{2}(\sigma_v - \sigma_h) = 0 \quad s = \frac{1}{2}(\sigma_v + \sigma_h) = 144\,kPa \quad s' = s - u = 74\ kPa$

effective stress, point $A' = (t', s') = (0, 74)$
total stress, point $A = (t', s) = (0, 144)$

during embankment construction $\Delta\sigma_h = \frac{\Delta\sigma_v}{4}$, hence
$\Delta t = \frac{1}{2}(\Delta\sigma_v - \Delta\sigma_h) = \frac{3}{8}\Delta\sigma_v \quad \Delta s = \frac{1}{2}(\Delta\sigma_v + \Delta\sigma_h) = \frac{5}{8}\Delta\sigma_v$
slope of total stress path, TSP, $\frac{\Delta t}{\Delta s} = \frac{3}{5}$
note that $\Delta\sigma_v = \Delta s + \Delta t \quad \Delta\sigma_h = \Delta s - \Delta t$

Untie yield occurs, the effective stress path (ESP) is vertical (elastic behaviour + undrained conditions). Yield occurs when the ESP meets the CSL ($t' = s' \sin\varphi'_{cs}$)

$\Delta s' = 0 \quad \Delta t = 74 \times \sin 23° = 28.9$ kPa $\quad \Delta s = \frac{5}{3}\Delta t = 48.2$ kPa

$s_Y = 144 + 48.2 = 192.2$ kPa $\quad \Delta\sigma_{vY} = \Delta s + \Delta t = 77.1$ kPa

pore water pressure at yield

$\therefore \quad u_Y = s_Y - s'_Y = 192.2 - 74 = 118.2$ kPa

$\therefore \quad$ safety factor at yield

$F = s_u/t = 100/28.9 = 3.46$ [40%]

(b) If the height of the embankment is raised to 1.5 H_Y, then the vertical stress will increase by a further $\Delta\sigma_v = \frac{1}{2}\Delta\sigma_{vY} = 38.55$ kPa

$$\Delta\sigma_v = \Delta s + \Delta t = \Delta s + \frac{3}{5}\Delta s = \frac{8}{5}\Delta s$$

$$\Delta s = \frac{5}{8}\Delta t = \frac{5}{8} 38.55 = 24.1 \text{ kPa}$$

$$\Delta t' = \Delta t = \frac{3}{5}\Delta s = \frac{3}{5} 24.1 = 14.5 \text{ kPa}$$

$$t_{eoc} = t'_{eoc} = t_Y + \Delta t = 28.9 + 14.5 = 43.4 \text{ kPa}$$

The total stress at the end of construction, eoc, is at point B = (216.3, 43.4)

$s_{eoc} = s_Y + \Delta s = 192.2 + 24.1 = 216.3$ kPa

The effective stress travels along the CSL to a value $s'_{eoc} = t_{eoc}/\sin\varphi'_{cs} = 43.4/\sin 23 = 111$ kPa

So the coordinates of point B' are:

$B' = (111, 43.4)$

The pore water pressure at the eoc is:

$$u_{eo} = s_{eoc} - s'_{eoc} = 216.3 - 111 = 105.3 \text{ kPa}$$

After dissipation of excess pore water pressures the pwp will go back to the initial value

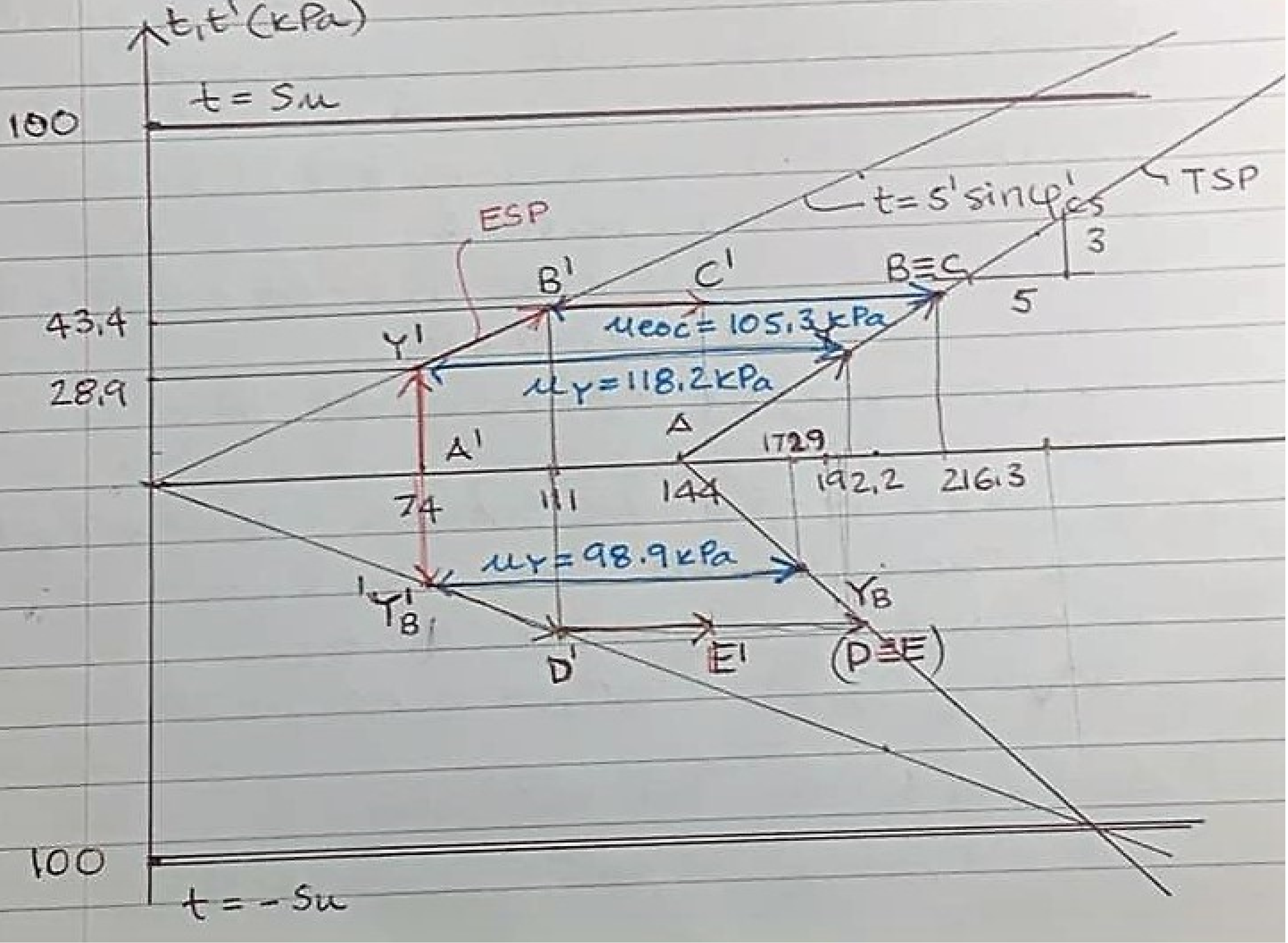
$$u_F = 70 \text{ kPa}$$

hence, $\Delta u = u_F - u_{eoc} = 70 - 105.3 = -35.3$ kPa and the effective stress point will move to C', while the total stress point does not move (C ≡ B)

$$s'_F = s_F - u_F = s_{eoc} - u_F = 216.3 - 70 = 146.3 \text{ kPa}$$

$$C' = (146.3, 43.4)$$

$$\varphi'_{mob} = \sin^{-1}\left(\frac{43.4}{146.3}\right) = 17.2° < \varphi_{cs}$$ [40%]



(c) Before embankment construction, the stress state at point B is the same as at soil element A.

Affective stress $A' = (0, 74)$
total stress $A = (0, 144)$

During embankment construction the vertical stress at soil element B will remain constant

$\Delta\sigma_v = 0$ therefore

$$\Delta t = \frac{1}{2}(\Delta\sigma_v - \Delta\sigma_h) = -\frac{1}{2}\Delta\sigma_h \qquad \Delta s = \frac{1}{2}(\Delta\sigma_v + \Delta\sigma_h) = \frac{\Delta\sigma_h}{2}$$

and the slope of the total stress path TSP will be $\frac{\Delta t}{\Delta s} = -1$

so soil element B will be loaded in extension under a constant vertical stress and an increasing horizontal stress

until yielding occurs the effective stress path will be vertical

$\Delta s' = 0 \quad \Delta t = -28.9$ kPa $\quad \Delta s = -\Delta t = 28.9$ kPa

$s_Y = 144 + 28.9 = 172.9$ kPa

The pore water pressure at yield will be

$u_Y = s_Y - s'_Y = 172.9 - 74 = 98.9$ kPa

note that $\Delta\sigma_v = 0 \quad \Delta\sigma_h = 57.8$ kPa

If raised by a further $0.5H_Y$

$\Delta\sigma_v = \frac{1}{2}\Delta\sigma_{vY} = 0 \qquad \Delta\sigma_h = \frac{1}{2}\Delta\sigma_{hY} = 28.9$

$\Delta t = -14.45$ kPa $\quad \Delta s = 14.45$ kPa

$\Delta s' = \Delta t' / \sin\varphi'_{cs} = 14.45 / \sin 23 = 36.98$

Q3

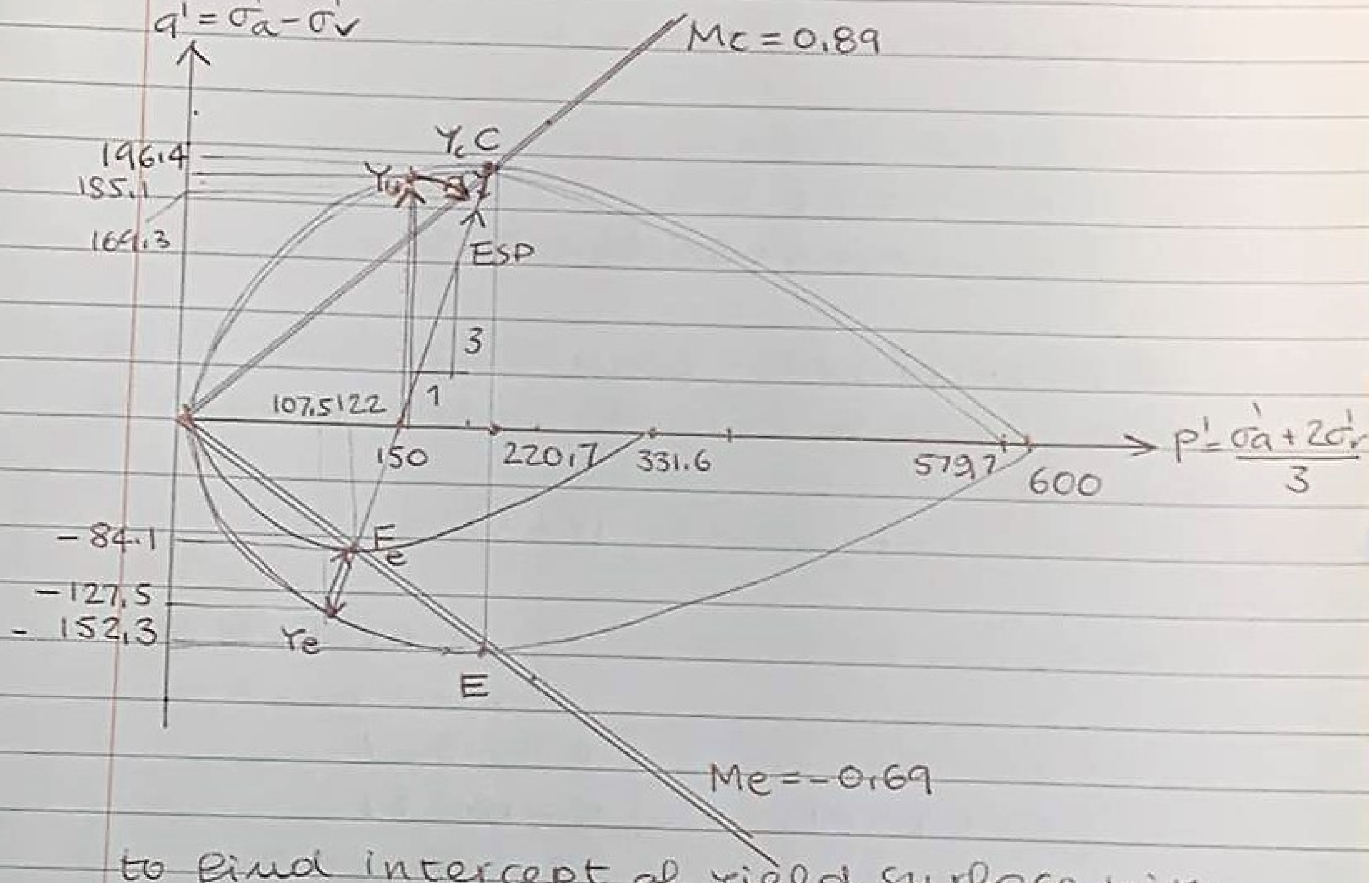
(a) NC to $p' = 600$ kPa

$v = N - \lambda \ln p' = \Gamma + \lambda - \kappa - \lambda \ln p' =$

$2.759 + 0.161 - 0.062 - 0.161 \ln 600 = 1.828$

swell back to $p' = 150$ kPa

$v = 1.828 + 0.062 \ln \frac{600}{150} = 1.914$



to find intercept of yield surface with critical state line, points C and E, solve

$$\begin{cases} q/p' = M_{c,e} \\ q/p' = M_{c,e} \ln \dfrac{p'_c}{p'} \end{cases}$$

$p' = \dfrac{p'_c}{\exp(1)} = 220.7$ kPa

$q = M_{c,e}\, p' =$ 196.4 kPa / −152.3 kPa

$C = (220.7,\ 196.4)$

$E = (220.7,\ -152.3)$

[20%]

(b) $\frac{\Delta q'}{\Delta p'} = \frac{\Delta \sigma'_a}{\Delta \sigma'_a / 3} = 3$

equation of ESP $q = -450 + 3p'$
to find yield point, solve:

$$\begin{cases} q = -450 + 3p' \\ q = 0.89\, p' \ln \frac{p'_c}{p'} \end{cases} \qquad -450 + 3p' = 0.89 p' \ln \frac{600}{p'}$$

$$\begin{cases} p' = 215.46 \text{ kPa} \\ q = -450 + 3 \times 215.46 = 196.38 \text{ kPa} \end{cases}$$

ESP intercepts camclay yield surface very close to critical state, on the dry side.

$$Y = (215.46,\ 196.38)$$

To find critical state point, F, solve

$$\begin{cases} q = -450 + 3p' \\ q = 0.89 p' \end{cases} \qquad \begin{cases} p' = 213.27 \\ q = 189.81 \end{cases}$$

$$F = (213.27,\ 189.81)$$

This corresponds to a (only very slightly) smaller value of p'_c

$$M_c = M_c \ln \frac{p'_c}{p'} \qquad p_c = \exp(1) \times 213.27 = 579.$$

To find specific volume @ yield

$$v_Y = N - \lambda \ln 600 + \kappa \ln \frac{600}{215.46} =$$

$$2.858 - 0.161 \ln 600 + 0.062 \ln \frac{600}{215.46} = 1.892$$

and at critical state

$$v_F = N - \lambda \ln p'_c + \kappa \ln \frac{p'_c}{p'} =$$

$$= 2{,}858 - 0{,}161 \ln 579.7 + 0{,}062 \ln \frac{579.7}{213{,}3} =$$

$$= 1{,}896$$

or, which is the same:

$$v_F = \Gamma - \lambda \ln 213{,}3 = 2{,}759 - 0{,}161 \ln 213{,}3 = 1{,}896$$

(c) to find yield point, solve

$$\begin{cases} q = -450 + 3p' \\ q = -0{,}69\, p' \ln \frac{p'_c}{p'} \end{cases} \Rightarrow \quad p' = 107{,}5 \text{ kPa} \quad q = -127{,}5 \text{ kPa}$$

$$Y = (107{,}5, -127{,}5)$$

to find critical state point, F, solve

$$\begin{cases} q = -450 + 3p' \\ q = -0{,}69 p' \end{cases} \Rightarrow \quad p' = 122 \text{ kPa} \quad q = -84{,}1 \text{ kPa}$$

$$F = (122, -84.1)$$

This corresponds to a new value of p'_c

$$M\eta = M\eta \ln \frac{p_c}{p'} \qquad p'_c = \exp(1) \times 122 = 331{,}6 \text{ kPa}$$

Specific volume at yield:

$$v_Y = N - \lambda \ln p'_c + \kappa \ln \frac{p_c}{p'} = 2{,}858 - 0{,}161 \ln 600 + 0{,}062 \ln \frac{600}{107{,}5}$$

$$= 1{,}935$$

and at critical state

$$v_F = \Gamma - \lambda \ln 122 = 2{,}759 - 0{,}161 \ln 122 = 1{,}986$$

(a) The effective stress path before yield will be vertical, $\Delta p' = 0$ (elastic + undrained) so the deviatoric stress at yield is:

$$q_Y = M_c p' \ln \frac{p'_c}{p'} = 0.89 \times 150 \times \ln \frac{600}{150} = 185.1 \text{ kPa}$$

and the yield point has coordinates

$$Y_u = (150, 185.1)$$

The deviator stress at failure can be found from:

$$q_F = q_{cs} = M_c p'_{cs} = M_c e^{\frac{\Gamma - v}{\lambda}} = 0.89 e^{\frac{2.759 - 1.9}{0.161}} =$$

$$= 169.3 \text{ kPa} \qquad p_F = q_F / M_c = 190.2 \text{ kPa}$$

and the point at failure has coordinates

$$F_u = (190.2, 169.3)$$

Assuming there is no back pressure the total stress path has equation

$$q = -450 + 3p$$

therefore the value of p at any q is equal to

$$p = \frac{q + 450}{3}$$

at yield $p_Y = \frac{185.1 + 450}{3} = 211.7$ kPa

and the pore pressure is $u = p_Y - p'_Y =$
$= 211.7 - 150 = 61.7$

at failure $p_F = \frac{169.3 + 450}{3} = 206.4$ kPa

and the pore pressure is $u = 206.4 - 190.2 = 16.$
k

Q4

(a) $\gamma = \frac{G_s + e}{1+e}\gamma_w = \frac{2{,}65+0{,}7}{1{,}7} \times 9{,}8 = 19.3 \text{ kN/m}^3$

$\sigma = (1\times18 + 3\times19.3)\cos^2 24^\circ = 63.3 \text{ kPa}$

$\tau = (1\times18 + 3\times19.3)\sin 24^\circ \cos 24^\circ = 28.2 \text{ kPa}$

$u = 3\times9.8\times\cos^2 24^\circ = 24.5 \text{ kPa}$

$\sigma' = \sigma - u = 38.8 \text{ kPa}$

$\varphi_{mob} = tg^{-1}\left(\frac{\tau}{\sigma'}\right) = tg^{-1}\left(\frac{28.2}{38.8}\right) = 36^\circ$

[30%]

(b) $I_D = \frac{0{,}85-0{,}7}{0.85-0{,}4} = 0{,}33$

$I_C = \ln \frac{20000}{38.8} = 6{,}25$

$I_R = 0{,}33\times6.25 - 1 = 1.06$

$\varphi_{peak} = \varphi_{cs} + 5 I_R = 36 + 5\times1.06 = 41.3^\circ$

The slope has a safety factor

$F = \frac{tg\,\varphi_{peak}}{tg\,\varphi_{mob}} = \frac{0{,}88}{0{,}73} = 1{,}2$

relative to the peak friction angle but is only marginally stable relative to the critical state friction angle

$F_{cs} = \frac{tg\,\varphi_{cs}}{tg\,\varphi_{mob}} \simeq 1$

(c) to fail the slope at the sandy soil/sand stone interface it must be

$\varphi_{mob} = \varphi_{peak}$

both φ_{mob} and φ_{peak} depend on σ'

which, in turn depends on u

For the sandy soil / sand stone interface

σ'	φ_{mob}	I_C	I_R	φ_P
38.8	36	6.25	1.06	41.3
32	41.4	6.44	1.11	41.6
31	42.3	6.47	1.14	41.7
→ 31.5	41.84			41.65

$\sigma' = 31.5$ kPa $\quad u = 63.3 - 31.5 = 31.8$ kPa

$u = h_w \times 9.8 \times \cos^2 24$

$$h_w = \frac{31.8}{9.8 \times \cos^2 24} = 3.9 \text{ m}$$

almost to ground surface

at contact between cover soil and sandy soil

$\sigma = 1 \times 18 \times \cos^2 24° = 15.02$ kPa
$\tau = 1 \times 18 \times \sin 24° \times \cos 24° = 6.69$ kPa
$u = \cancel{3.69}\ 0.9 \times 9.8 \times \cos^2 24 = 7.36$ kPa
$\sigma' = 7.66$ kPa

$$\varphi_{mob} = \tan^{-1}\left(\frac{6.69}{7.66}\right) = 41.13°$$

$\varphi_{peak} \simeq 44°$ ok

The deeper surfaces are more critical because their peak strength is less whereas the mobilised angle of friction