Version DJC/5

EGT1
ENGINEERING TRIPOS PART IB

Monday 1 June 2015   9 to 11

Paper 1

MECHANICS

*Answer not more than **four** questions.*

*Answer not more than **two** questions from each section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

*Answer not more than **two** questions from this section*

1 Figure 1 shows a rigid beam AB of length L and constant mass per unit length ρ supported at an angle of 45° to the horizontal by two long, vertical, light and inextensible cables, one at each end of the beam. The beam is stationary when suddenly the cable that supports end A breaks. For the instant immediately after it breaks:

(a) show that the horizontal acceleration of the centre of the beam is zero; [3]

(b) derive expressions for the instantaneous:

(i) angular acceleration of the beam; [3]

(ii) vertical acceleration of the centre of the beam; [3]

(iii) tension in the cable that supports end B of the beam; [3]

(c) derive an expression for the instantaneous shear force in the beam in terms of distance along the beam and hence find the position of the maximum magnitude of bending moment. [13]

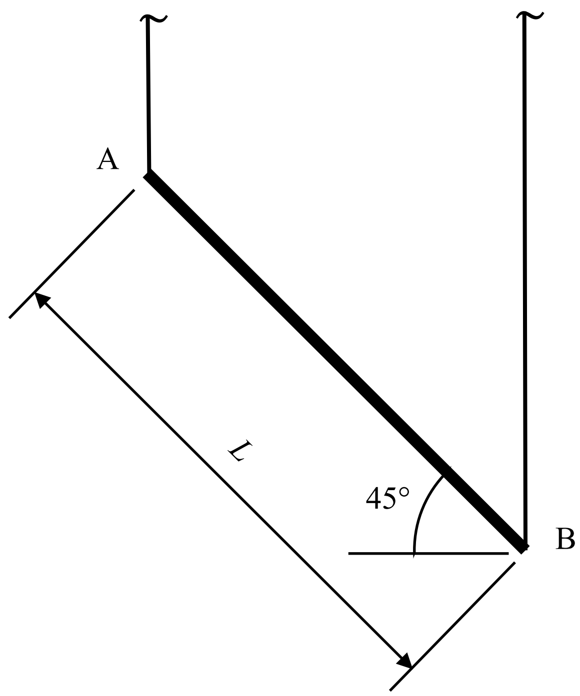

Fig. 1

2 A box of mass m uniformly distributed across a rectangular cross-section is moved in a vertical plane using an hydraulic actuator and a four-bar linkage ABCD as shown in Fig. 2. The actuator extends at a constant rate V and at the instant shown the four-bar linkage forms a rectangle. Friction torque Q acts at each joint in the four-bar linkage. There is no friction acting at the joints connecting the actuator to the four-bar linkage. For the instant shown in the figure:

(a) by drawing a velocity diagram (suggested scale V = 100 mm), or otherwise, determine:

(i) the velocity of point C; [2]

(ii) the angular velocities of the actuator AC and the link CD; [4]

(b) by drawing an acceleration diagram (suggested scale V^2/L = 100 mm), or otherwise, determine the linear and angular accelerations of the box; [13]

(c) use the method of virtual power to determine the force in the actuator. [6]

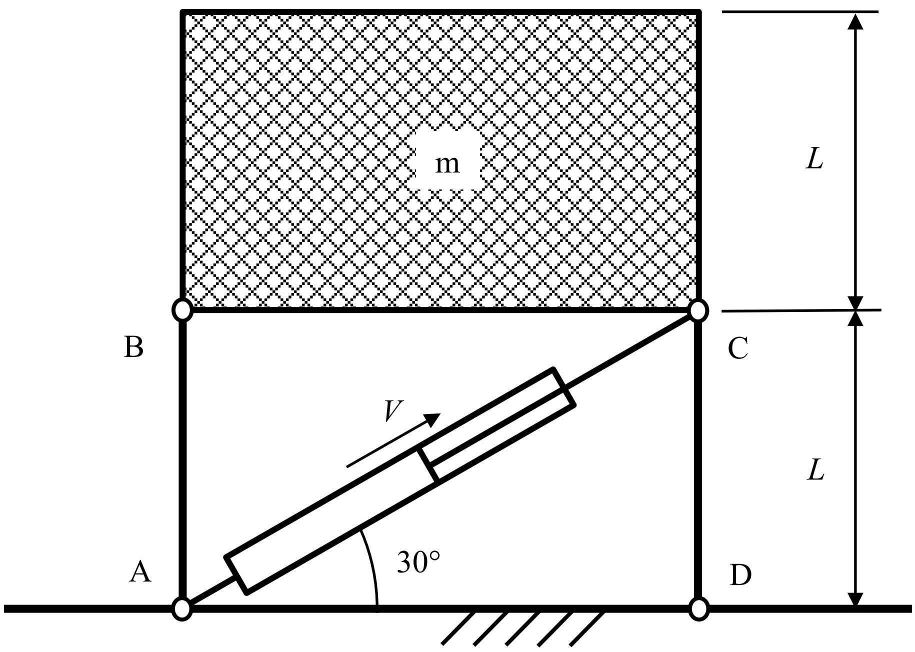

Fig. 2

3 The end elevation of a racing car is shown in Fig. 3a. The car has track width $2a$, mass m and centre of mass at height h. The car is following a horizontal circular path of radius R at constant speed U. The car is fitted with an energy storage device consisting of a flywheel with polar moment of inertia J and spin speed ω. The layout of the device is shown in Fig. 3b. The flywheel is a solid circular disc with radius r and thickness $2b$, and spins on bearings at A and B, distance $4b$ apart. The spin axis of the flywheel is parallel to the wheels' axes of rotation. The gyroscopic torques of the wheels are negligible.

(a) If the flywheel spin speed ω is zero, derive an expression for the vehicle speed above which the vehicle will roll over (that is, the inside wheels lose contact with the road). Assume that the vehicle does not slide before rollover. [5]

(b) Derive an expression for the flywheel spin speed ω that must be set if the left and right vertical wheel forces are to remain equal to each other. Specify whether the flywheel should spin in the same direction or opposite direction to wheel rotation and explain why. [10]

(c) The flywheel is out of balance. At spin speed ω the amplitudes of dynamic force applied to the bearings by the flywheel are found to be P at bearing A and $2P$ at bearing B, exactly in phase with each other. Derive expressions for the balance mass or masses that must be added to the flywheel at radius r on outside faces C and D to achieve dynamic balance. State the angular orientation of the mass(es) with respect to the direction of the original dynamic forces applied to the bearings by the flywheel. [10]

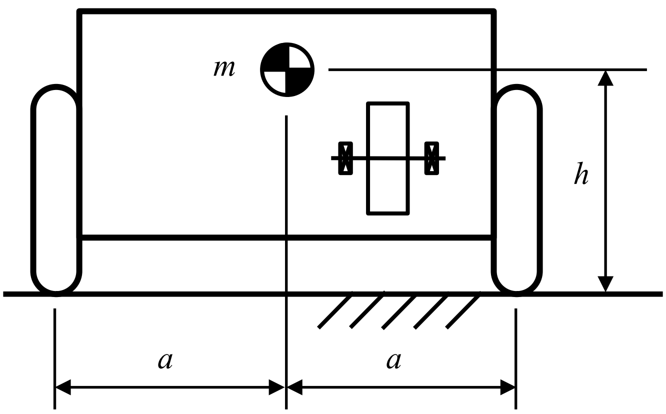

Fig. 3a

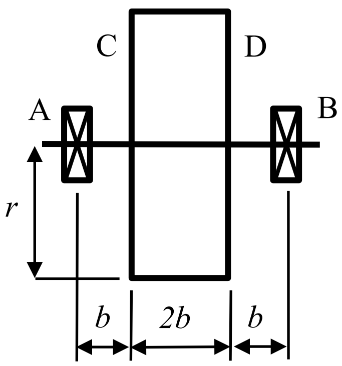

Fig. 3b

SECTION B

Answer not more than ***two*** *questions from this section.*

4 A lorry of height $4a$ and width $3a$ slides sideways with velocity v and impacts a rigid barrier which has height a, as shown in Fig. 4. There is no rebound and no sliding at the point of impact. The lorry is represented as a rigid body with uniformly distributed mass.

(a) Find the angular velocity of the lorry immediately after impact, in terms of v and a. [7]

(b) Find the maximum value of v such that the lorry will not roll completely over the barrier. [11]

(c) A lorry of height 3.2 m, width 2.4 m and mass 20×10^3 kg slides sideways and impacts a barrier which is 0.8 m high. Find the maximum sideways velocity of the lorry such that it will not roll completely over the barrier. Find also the energy that must be absorbed in the impact if the lorry collides with the barrier at this velocity. [7]

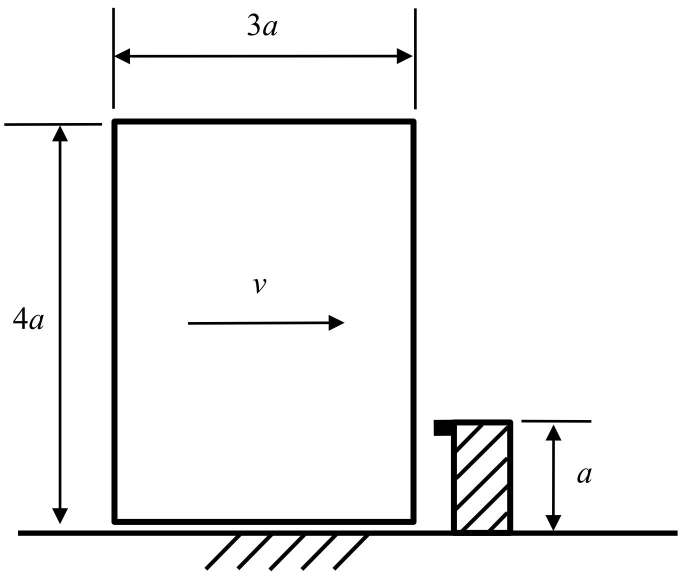

Fig. 4

5 A small aeroplane is 'looping the loop' by flying around a circular path of radius 200 metres in the vertical plane as shown (not to scale) in Fig. 5. At the instant shown the aeroplane is at the lowest point of the circle and has a speed of 80 m s^{-1} that is increasing at a rate of 3 m s^{-2}.

(a) The pilot of the aeroplane has a mass of 80 kg and is supported entirely by the pilot's seat. Find the magnitude and direction of the force that is exerted on the seat at this instant. [6]

(b) The propeller is at the front of the aeroplane, has a diameter of 3 m and is rotating at a steady speed of 3342 revolutions per minute relative to the aeroplane in a clockwise direction as viewed by the pilot. At the instant shown one blade of the propeller is exactly horizontal and is pointing out of the paper in Fig. 5 (that is, in the +**k** direction). Express the absolute velocity of the tip of this blade as a vector in (**i**,**j**,**k**) space. [6]

(c) Express the absolute acceleration of the blade tip described in part (b) as a vector in (**i**,**j**,**k**) space. [13]

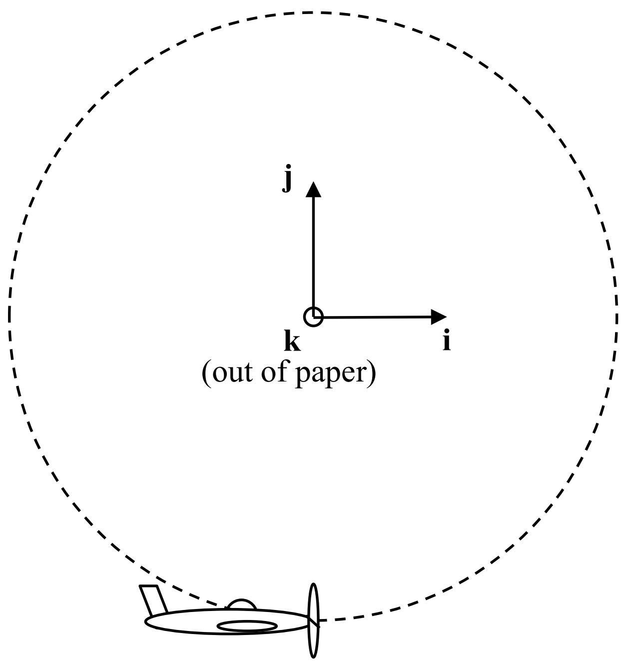

Fig. 5

6 A firm of novelty manufacturers is planning to make a rocking figure by fixing a light hollow cone made of paper to a solid hemispherical base, as shown in Fig. 6. In order to reduce the frequency at which the figure rocks, it is proposed to add a short solid cylindrical section to the hemispherical base, as shown in Fig. 6(a). The hemisphere and the cylinder have the same, uniform density.

(a) If the radius of the hemisphere is a, what is the maximum length b of the cylinder such that the figure will not fall over when placed on a flat surface? [8]

(b) The firm decides instead to reduce the frequency of rocking by pressing a small pellet of metal into the centre of the hemisphere's base circle, as shown in Fig. 6(b). The mass of the pellet is the same as that of the hemisphere, and its moment of inertia is negligible. What is the frequency at which the figure will now rock, if $a = 0.02$ m? [17]

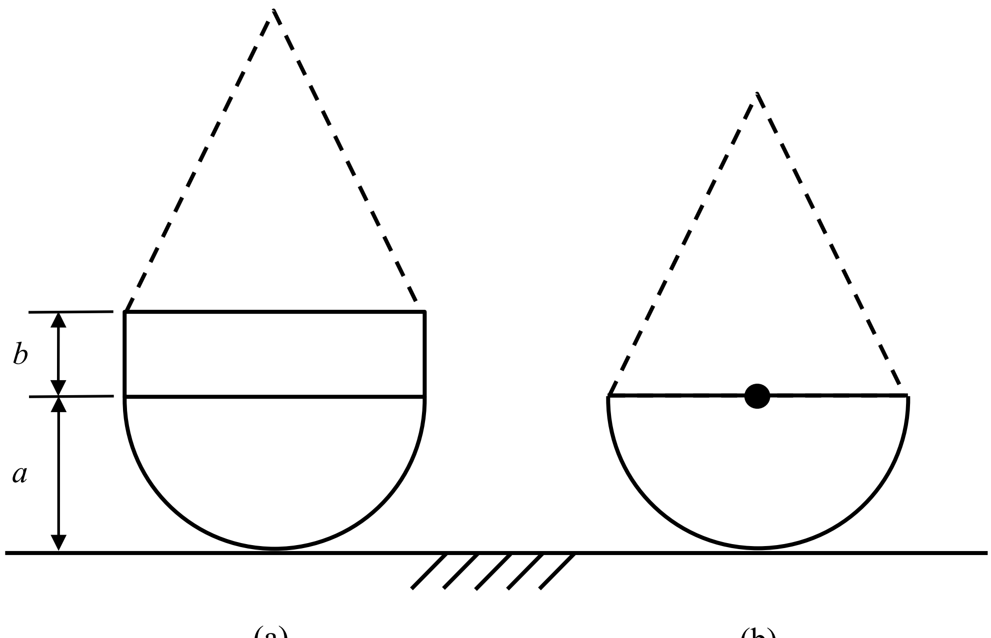

Fig. 6

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