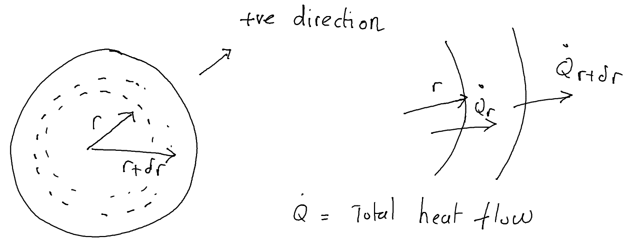
1(a)



Considering energy flows into and out of shell.

$$0 = \underbrace{\dot{Q}_r}_{\text{inflow of heat}} - \underbrace{\dot{Q}_{r+dr}}_{\text{outflow of heat}} + \underbrace{4\pi r^2 dr\, H}_{\text{heat generation}}$$

$$\dot{Q}_{r+dr} = \dot{Q}_r + \frac{\partial(\dot{Q})}{\partial r} dr$$

$$\Rightarrow \quad 0 = \cancel{\dot{Q}_r} - \left[\cancel{\dot{Q}_r} + \frac{\partial}{\partial r}\dot{Q}\, dr\right] + 4\pi r^2 dr\, H$$

$$0 = -\frac{\partial \dot{Q}}{\partial r} + 4\pi r^2 H$$

Also $$\dot{Q} = -4\pi r^2 \lambda \frac{dT}{\partial r}$$

$$\Rightarrow \quad 0 = -\frac{\partial}{\partial r}\left[-4\pi r^2 \lambda \frac{\partial T}{\partial r}\right] + 4\pi r^2 H$$

$$\therefore \Rightarrow \quad 0 = \lambda \frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + r^2 H$$

$$\therefore \quad 0 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{H}{\lambda}$$

(b) (i)

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) + \frac{H}{\lambda} r^2 = 0$$

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = -\frac{H}{\lambda}r^2$$

Integrating

$$r^2\frac{dT}{dr} = \int \frac{H}{\lambda}r^2 dr + C$$

$$r^2\frac{dT}{dr} = -\frac{\frac{H}{\lambda}r^3}{3} + C$$

At $r = 0, r^2\frac{dT}{dr} = 0$ (makes the constant of integration zero, i.e. don't need to enforce the central boundary condition.

$$r^2\frac{dT}{dr} = -\frac{H}{3\lambda}r^3$$

$$\boxed{\frac{dT}{dr} = -\frac{H}{3\lambda}r}$$

(b) ii) Starting from

$$\frac{dT}{dr} = -\frac{H}{3\lambda}r$$

Integrating gives the temperature profile as

$T(r) - T(a) = -\frac{H}{6\lambda}(r^2 - a^2)$So the temperature difference between the centre and the surface is

$$\boxed{T(0) - T(a) = \frac{H}{6\lambda}(a^2)}$$

(c) i)

The flux of heat leaving the particle must match heat flux through the surrounding fluid to the bulk. The total heat flow out the particle is

$$\dot{Q} = -4\pi a^2\lambda\frac{dT}{dr} = 4\pi a^3\frac{H}{3} = \frac{4\pi a^3}{3}H$$

Giving the heat flux at the surface as

$$\dot{q} = \frac{\frac{4\pi a^3}{3}H}{4\,\pi a^2} = \frac{Ha}{3}$$

(alternatively, use part b(i) and note that at the surface $\dot{q} = -\lambda\frac{dT}{dr}_a = \frac{Ha}{3}$)

Since $Nu = \frac{2ah}{\lambda_{ext}}$ where h is the heat transfer coefficient and λ_{ext} is the thermal conductivity of the argon flowing over the spheres.

$$q = \frac{Nu}{2a}\lambda_{ext}(T(a) - T(\infty)) = Ha/3$$

$$T(a) - T(\infty) = \frac{2Ha^2}{3\lambda_{ext}Nu}$$

Therefore the ratio of internal to external temperature difference is

$$\frac{T(0) - T(a)}{T(a) - T(\infty)} = \frac{\Delta T_{int}}{\Delta T_{ext}} = \frac{H(a^2)}{\frac{2Ha^2}{3\lambda_{ext}Nu}6\lambda}$$

$$\frac{\Delta T_{int}}{\Delta T_{ext}} = \frac{Nu\lambda_{ext}}{4\lambda}$$

$$\frac{\Delta T_{int}}{\Delta T_{ext}} = 0.01 \Rightarrow Nu = 0.01 \times 4 \times 700 = 28$$

$$Bi = \frac{hs}{\lambda} = \frac{Nu\lambda_{ext}}{2a\lambda} \times \frac{\frac{4}{3}\pi a^3}{4\pi a^2} = \frac{Nu\lambda_{ext}}{6\lambda} = \frac{28}{6 \times 700} = 0.0067$$

i.e. very small – lumped heat capacity model is appropriate.

(c)(ii)

Since the lumped heat capacity model can be used, it is straightforward to write the time dependant equation for the temperature of a sphere.

$$\frac{4}{3}\pi a^3 \rho c \frac{dT}{dt} = -4\pi a^2 h(T - T_\infty) + \frac{4}{3}\pi a^3 H$$

Simplifying and letting $\theta = T - T_\infty$

$$\rho c \frac{d\theta}{dt} = -\frac{3h}{a}\theta + H$$

Or

$$\frac{\rho c a}{3h}\frac{d\theta}{dt} + \theta = +\frac{a}{3h}H$$

$$\tau \frac{d\theta}{dt} + \theta = +\frac{a}{3h}H$$

Which is a standard first order ODE, whose transient response is determined by the solution to the homogenious equation and the time constant

$$\tau = \frac{\rho c a}{3h}$$

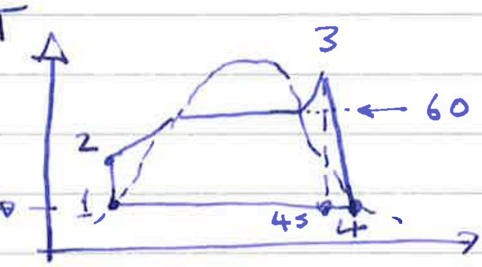
Since $h = \frac{Nu\lambda_{ext}}{2a}$

$$\tau = \frac{2\rho c a^2}{3Nu\lambda_{ext}}$$

$$\boxed{\tau = \frac{2 \times 1000 \times 1000 \times (0.01)^2}{3 \times 28 \times (\frac{70}{700})} = 238s}$$

It would take about 3 time constants to have almost reached the steady state.

Rankine cycle:



a) (i) $h_1 = 167.5$ kJ/kg

Reversible pump: $\Delta h = \int v\,dp \simeq \bar{v}\,\Delta p = 6.97$ kJ/kg

using $\bar{v} = \frac{1}{2}(0.001008 + 0.001319)$

$\Rightarrow h_2 = 174.5$ kJ/kg

Tables (or chart): $h_3 = 3178.2$ kJ/kg $\Rightarrow q_{boiler} = \underline{\underline{3003.7}}$

(ii) By definition, $s_{4s} = s_3 = 6.543$ kJ/kgK (tables/chart)

On 40°C sat'n line, $s_f = 0.572$, $s_g = 8.256 \Rightarrow x = 0.7771$

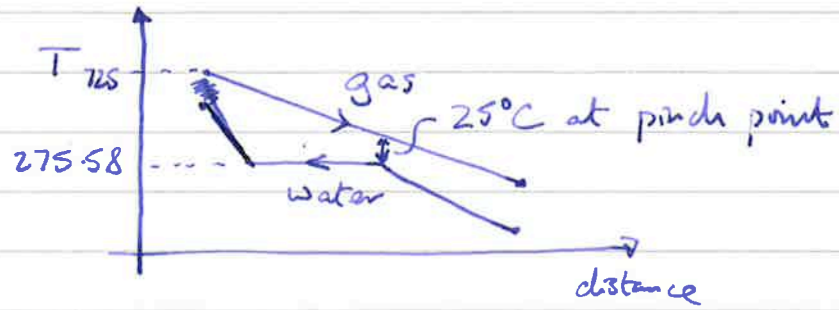
Hence $h_{4s} = h_1 + x \underset{\uparrow 2406.0}{h_{fg}} = 2037.1$

Turbine work output $= \eta_{is}(h_3 - h_{4s}) = \underline{\underline{1027.0}}$ kJ/kg

(iii) $$\eta_{th} = \frac{\text{turbine work} - \text{pump work}}{q_{boiler}}$$

$$= \frac{1027.0 - 7.0}{3003.7} = \underline{\underline{0.340}}$$

b) Boiler diagram



(i) Water enthalpy gain from pinch point to turbine inlet is:

$\dot{m}_w (h_3 - h_p)$ with $h_p = h_f(60\text{ bar}) = 1213.9$ kJ/kg

This is equal to heat given up by gas: $\dot{m}_g c_p (725 - 300.58)$

Hence $\dot{m}_g / \dot{m}_w = (3178.2 - 1213.9) \times 10^3 / [10^3 \times 424.42]$

$= 4.6282 = \underline{\underline{4.63}}$ (3sf)

(ii) $(\Delta T)_{gas} = \dfrac{\dot{m}_w (h_3 - h_4)}{\dot{m}_g c_p} = \dfrac{3003.7}{4.6282} = 649.0$

Hence outlet temp is $725 - 649.0 = \underline{\underline{76°C}}$

c) (i) $(\Delta \dot{H})_{gas} = 3003.7$ $(\Delta \dot{S})_{gas} = \dot{m}_g c_p \ln\left(\dfrac{T_{in}}{T_{out}}\right) = +4.8605$ kJ/Ks

$\therefore \Delta \dot{B} = \underline{1555.0}$ kJ/s for $\dot{m}_w = 1$ kg/s

(ii) Availability loss gives ideal (reversible) work output that is attainable in association with cooling of gases to T_0.
Hence it differs due to: (i) $T_1 > T_0$; and (ii) irreversibilities. Latter arise from:

- heat transfer across finite temperature difference in boiler;
- cycle irreversibility (in this case, due to turbine only).

a) (i) $CH_4 + 1.5 \times 2\left(O_2 + \frac{79}{21} N_2^*\right) \rightarrow CO_2 + 2H_2O + O_2 + 11.286 N_2^*$

(ii) SFEE for gas passing thro' boiler:

$$Q = (\Delta H_0)_{reaction} + (\Delta H)_{products}$$

$$(\Delta H_0)_{reaction} = -50.01 \text{ MJ/kg} \times 16 \text{ kg/kmol} = -800.16 \text{ MJ/kmol}$$

Products	No. kmols	$h_{25°C}$ (MJ/kmol)	h_{500K} (MJ/kmol)	ΔH
CO_2	1	9.37	17.67	8.30
H_2O	2	9.90	16.82	13.84
O_2	1	8.66	14.74	6.08
N_2^*	11.286	8.67	14.58	66.70
				94.92

$\therefore$ per kmol of CH_4, $Q = -800.16 + 94.92 = -705.24$

This is a heat loss for the gas, so heat exchanger extracts 705 MJ per kmol CH_4

b) (i) $CH_4 + \frac{3}{2} O_2 \rightarrow CO + 2H_2O$

(ii) $CO + \frac{1}{2} O_2 \rightarrow CO_2$

(iii) For (ii), the value can be derived directly from data book figures: $CV = 10.1 \times 28 = 282.8$ MJ/kmol CO

For (i), need to observe that (i) and (ii) together represent stoichiometric combustion of methane.

Question number 3 Sheet number 2

Hence (per kmol of CH_4): $(CV)_{(i)} + (CV)_{(ii)} = 800.16$ MJ

$\therefore (CV)_i = 800.16 - 282.8 = \underline{517.4}$ MJ/kmol CH_4

c) (i) $CH_4 + 1.75\left(O_2 + \frac{79}{21}N_2^*\right) \rightarrow 2H_2O + xCO_2 + yCO + 6.58N_2^*$

Now $x+y = 1$ for carbon balance

Oxygen: $1.5 = 2x + (1-x) = 1+x$, so $x = 0.5$
$y = 0.5$

(ii) The energy-releasing reactions (per kmol of CH_4) are:

$\frac{1}{2}\left[CH_4 + \frac{3}{2}O_2 \rightarrow CO + 2H_2O\right]$ CV 517.36 MJ/kmol

and $\frac{1}{2}\left[CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O\right]$ CV 800.16 MJ/kmol

Hence the combustion reaction now has CV

$0.5 \times 517.36 + 0.5 \times 800.16 = \underline{658.8}$ MJ/kmol CH_4

OR $CH_4 + \frac{3}{2}O_2 \rightarrow CO + 2H_2O$

and $\frac{1}{2}\left[CO + \frac{1}{2}O_2 \rightarrow CO_2\right]$

with CV $517.36 + \frac{1}{2} \times 282.8 = 658.8$ MJ/kmol CH_4 again.

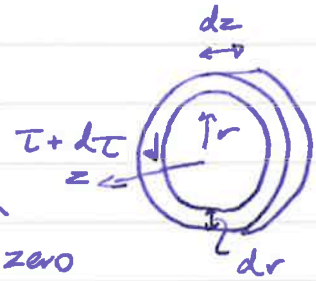
Question number 4 Sheet number 1

a) Rigid-body motion $\Rightarrow$ $u_\theta = \omega r$

Hence $\mu r \frac{d}{dr}\left(\frac{u_\theta}{r}\right) = \mu r \frac{d\omega}{dr} = \underline{\underline{0}}$

Physical explanation: motion involves no shearing between adjacent cylindrical surfaces.

b) (i) Consider a small ring element:



- No rate of change of angular momentum (steady flow) $\Rightarrow$ net torque must be zero
- No torque from cylindrical surfaces, as shown for this flow topology in part (a). [NB Torque is about z axis.]
- Hence $(\tau + d\tau - \tau)\, 2\pi r\, dr . r = 0$

$$d\tau = 0$$

But $d\tau = \frac{\partial \tau}{\partial z} dz$ for this geometry.

Hence $\tau = \tau(r)$, independent of z.

(ii) We have $u_\theta = \Omega r$, where Ω must now be a function of z.

Hence $\tau = \mu r \frac{d\Omega}{dz}$ = ~~const~~ fn(r) only

B/Ei

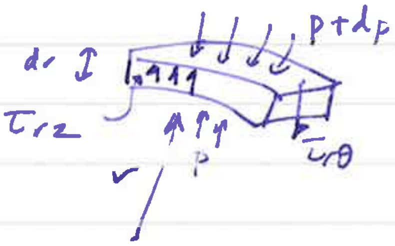
and thus $\Omega = \Omega_0 + \Omega_1 z$

To match the b.c.s at the discs, we have

$$\Omega_0 = \frac{\omega_1 + \omega_2}{2} \qquad \Omega_1 = \frac{\omega_2 - \omega_1}{h}$$

and hence $$u_\theta = r\left[\frac{\omega_2+\omega_1}{2} + \frac{\omega_2-\omega_1}{h} z\right]$$

(iii) Streamline normal equation deals with Navier II in radial direction:

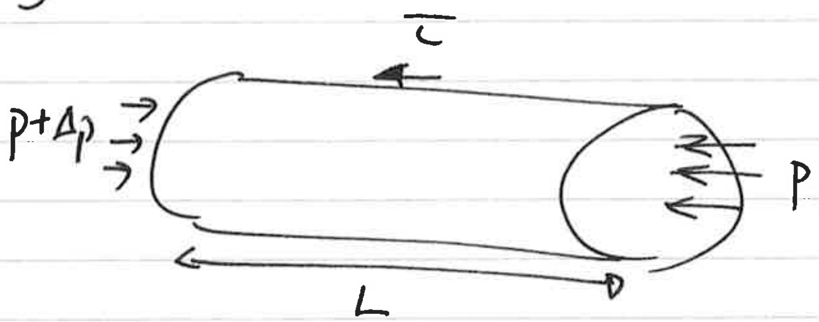


The relevant force contributions arise from: pressure p, shears $\tau_{r\theta}$, τ_{rz}

However, in this particular flow, there's no shearing motion that would give rise to $\tau_{r\theta}$, τ_{rz}, so the only force acting is due to p, which is the same as for inviscid flow.

N.B. Strictly speaking, should also consider the possibility of a normal viscous force, but this isn't expected.

a) Apply momentum eqn to pipe flow :



Fully-developed, incompressible flow $\Rightarrow$ no change in momentum flux between ends, so momentum eqn reduces to force equilibrium:

$$\Delta p \cdot \frac{\pi d^2}{4} = \tau \cdot \pi d L$$

i.e. $$\Delta p = 4\tau \frac{L}{d}$$

b)(i) Already defined : (τ), d

ρ : fluid density
V : fluid (mean) velocity, i.e. Vol. flow rate $/ (\pi d^2/4)$
ν : kinematic viscosity of the fluid
k : roughness height of the pipe material

Possible to summarise in terms of three variables only because of dimensional analysis:

$$\tau = fn(\underbrace{d, \rho, V, \nu, k}_{})$$ 5 variables, 3 dimensions, hence 2 dimensionless groups

$$\frac{\tau}{\frac{1}{2}\rho V^2} = fn\left(\frac{Vd}{\nu}, \frac{k}{d}\right).$$

b) (ii) First, note that this region is in the turbulent regime. Here the shear stresses arise from momentum transport via fluid eddies, rather than from molecular viscosity.

Eddy sizes (strictly those of the smallest) are in general determined by the Reynolds number and the relative roughness. However the region where c_f is only a function of k/d must correspond to eddy sizes that are independent of Re. Hence the interpretation is that the (smallest) eddy size is set purely by the roughness height.

c) (i) $Re = \frac{Vd}{\nu} = 1.63\times10^{6}$ $\quad \frac{k}{d} = \frac{0.05\times10^{-3}}{1} = 5\times10^{-5}$

Hence, from chart, $c_f = 0.003$

$$\tau = c_f . \tfrac{1}{2}\rho V^2 = 0.231 \text{ Pa}$$

$$\Delta p = 4\tau L/d = \underline{\underline{924}} \text{ Pa}$$

(ii) Compressor power will be proportional to $Q\,\Delta p$, and Q is unaffected by pipe diameter.

$$\Delta p = 2\rho V^2 c_f \frac{L}{d} \propto d^{-5} c_f .$$

If c_f is constant, power required drops as d increases.
In fact, c_f also varies due to decrease in Re, decrease in k/d.
e.g. Increase d to 2.5m, $Re = 6.54\times10^{5}$ $k/d = 2\times10^{-5}$ $c_f \simeq 3.3\times10^{-3}$
So power decreases by factor 89 ($= 2.5^{4.9}$); c_f influence trivial.

UNIVERSITY OF CAMBRIDGE

Question number 6 Sheet number 1

a) (i) Streamlines parallel ⇒ pressure uniform at plane A.

Incompressible, inviscid flow ⇒ Bernoulli applies

Hence, for any streamline, $p_A + \frac{1}{2}\rho V^2 = p_P$

i.e. V is same for all s/lines @ A.

(ii) Bounding streamline has $p = p_a$

Hence $p_A = p_a$, and $p_P - p_a = \frac{1}{2}\rho V^2$

b) (i) Static pressure at pipe exit is p_P (parallel s/lines again)

Hence stagnation pressure is $p_P + \frac{1}{2}\rho\left(\frac{Q}{A}\right)^2$

(ii) Plenum stagnation pressure is just p_P (no significant flow velocities).

∴ Pipe stagnation pressure is higher, by an amount $\frac{1}{2}\rho\left(Q/A\right)^2$.

Difference is accounted for by mixing loss as jet spreads out and slows downstream of pipe exit.

B/ER

c) (i) To support hovercraft, $p_p \cdot \frac{\pi D^2}{4} = W$

$$p_p = \frac{4W}{\pi D^2}$$

N.B. p_p is gauge here!

(ii) From (i) and b(ii), supply pipe stagnation pressure is:

$$p_p + \frac{1}{2}\rho\left(\frac{Q}{A}\right)^2 = \frac{4W}{\pi D^2} + \frac{1}{2}\rho\left(\frac{Q}{A}\right)^2$$ also gauge

Since this is gauge stagnation pressure @ compressor exit, it's equal to Δp_0, i.e.

$$\frac{4W}{\pi D^2} + \frac{1}{2}\rho\left(\frac{Q}{A}\right)^2 = C_0 - C_2 Q^2$$

$$Q^2\left[C_2 + \frac{\rho}{2A^2}\right] = C_0 - 4W/\pi D^2$$

$$Q^2 = \left[C_0 - 4W/\pi D^2\right] / \left[C_2 + \rho/2A^2\right]$$

(iii) We can only say something definite about plane A (or, more carefully, it's equivalent for the hovercraft flow).

Let this be at diameter D_A, with jet height h_A.

Continuity then gives $h_A = Q/(\pi D_A V)$, where V is fixed by the result from a)(ii).
Given the flow geometry in Fig. 3(a), this is a lower bound.