EGT1
ENGINEERING TRIPOS PART IB

Tuesday 2 June 2015 2 to 4

Paper 4

THERMOFLUID MECHANICS

Answer not more than ***four*** *questions.*

Answer not more than ***two*** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

Answer not more than *two* questions from this section.

1 In a pebble-bed nuclear power plant, the reactor is packed with spherical fuel particles of radius a. The heat released by the nuclear reaction within the particles is transferred to a stream of argon which flows over them. The rate of heat generation per unit volume within a particle is H, and the thermal conductivity of the sphere material is λ.

(a) By considering the conduction of heat into and out of a thin spherical shell of radius r and thickness dr, show that the steady-state temperature $T(r)$ within a fuel particle is governed by

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}T}{\mathrm{d}r}\right)+\frac{H}{\lambda}=0.$$ [5]

(b) In terms of a, H and λ, derive expressions for:

(i) the temperature gradient within the sphere, given that it is zero at the centre; [5]

(ii) the temperature difference between the centre and surface of the sphere. [5]

(c) The heat-transfer coefficient from the surface of the sphere to the argon is determined by the Nusselt number. The thermal conductivity of the sphere material is 700 times that of the argon.

(i) How small would the Nusselt number need to be to reduce the temperature difference within the sphere to 1% of that between the surface of the sphere and the bulk fluid? Comment on the value of the Biot number in this case. [5]

(ii) A sphere with $a = 0.01$ m, $H = 100$ W m^{-3}, $\lambda = 7$ W m^{-1}K^{-1}, heat capacity 1000 J kg^{-1}K^{-1} and density 1000 kg m^{-3} is initially at a temperature below the steady-state value. For the Nusselt number found in part (i), *estimate* the time scale over which the steady state would be attained. [5]

2 A Rankine cycle operates with water as the working fluid. Saturated liquid at 0.07385 bar is pumped (reversibly) to 60 bar and heated in the boiler tubes, leaving superheated at 400°C. The superheated steam passes through a turbine with an isentropic efficiency of 0.9, and then to a condenser operating at 0.07385 bar.

(a) Assuming that the mass flow rate of water around the cycle is 1 kg s^{-1}, calculate:

(i) the heat energy added in the boiler; [5]

(ii) the work output of the turbine; [6]

(iii) the thermal efficiency of the cycle. [2]

(b) The heat supply to the cycle comes from hot combustion gases which flow over the boiler tubes at constant pressure. The gases enter the boiler at 725°C, and flow in the opposite direction to the water in the tubes. They can be assumed to behave as a perfect gas with $c_p = 1000$ J $kg^{-1}K^{-1}$.

(i) The mass flow rate of the combustion gases is set so that they are always at least 25°C hotter than the water. What is the ratio of gas and water mass flows? [4]

(ii) What is the temperature of the combustion gases when they leave the boiler? [2]

(c) (i) Again assuming a 1 kg s^{-1} water mass flow rate, calculate the reduction in availability of the combustion gases as they cool. [2]

(ii) Explain why the work output of the cycle is smaller than the availability reduction calculated in part (i). [4]

Additional information:
Flows through pumps and turbines can be assumed to be adiabatic.
The availability is given by $b = h - T_0 s$, where $T_0 = 298$ K is the temperature of the environment.

3 (a) In an industrial boiler, methane (CH_4) is combusted with dry air. The air flow rate is 1.5 times that required for stoichiometric combustion. The air and methane enter the boiler at 25°C. Within the boiler, the gases react in a combustion chamber and then pass over a set of heat-exchanger tubes, before exiting at 227°C. The pressure is 1 bar throughout.

(i) Write down the chemical equation for the combustion process. [3]

(ii) How much heat is extracted from the combustion gases by the heat exchanger, per kmol of methane entering the boiler? [8]

(b) (i) What is the chemical equation for the reaction between methane and oxygen that produces only carbon monoxide and water? [2]

(ii) What further reaction is required to convert the carbon monoxide to carbon dioxide, without any other products? [2]

(iii) What are the calorific values, *per kmol of the respective fuels*, of the reactions in parts (i) and (ii)? [4]

(c) The air flow in the boiler of part (a) is reduced to 87.5% of that required for stoichiometric combustion. At the exit of the combustion chamber there is no hydrogen present.

(i) What is the chemical equation for the reaction that has taken place in the combustion chamber? [3]

(ii) What is the calorific value of this reaction, per kmol of methane entering the boiler? [3]

SECTION B
Answer not more than *two* questions from this section.

4 (a) A cylinder of incompressible fluid is in rigid-body motion, spinning about its axis with angular velocity ω. The dynamic viscosity of the fluid is μ, and its azimuthal velocity is denoted by u_θ. The azimuthal shear stress on a cylindrical surface at radius r from the axis is given by

$$\mu r \frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{u_\theta}{r}\right).$$

Show that this quantity is equal to zero, and explain physically why this is so. [5]

(b) A fluid coupling consists of a pair of spinning discs, with angular velocities ω_1 and ω_2, separated by a distance h (see Fig. 1). This gap is filled by an incompressible viscous fluid, which is prevented from escaping at the disc edges. The flow in a disc-shaped fluid layer at axial position z has the rigid-body form described in part (a). The layer's faces (*i.e.* the surfaces normal to z) experience azimuthal shear stresses given by

$$\tau = \mu \frac{\partial u_\theta}{\partial z}.$$

(i) Show that τ is independent of z. [6]

(ii) Find u_θ as a function of z and r. [8]

(iii) The streamline-normal equation of motion for an inviscid flow of this form is

$$\frac{\partial p}{\partial r} = \rho \frac{u_\theta^2}{r},$$

with p the pressure and ρ the density. Explain qualitatively, with the aid of sketches, why this equation also applies for the fluid-coupling flow. [6]

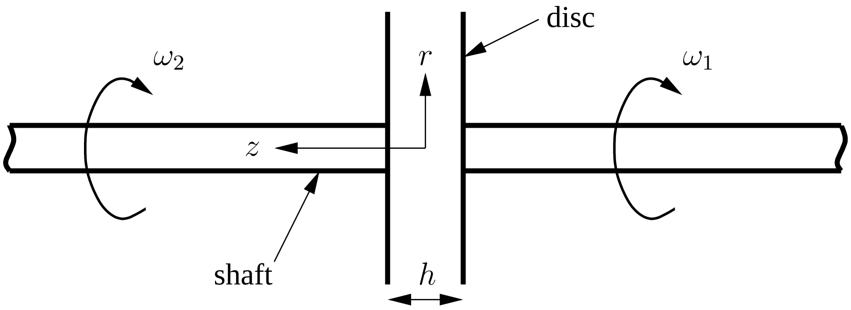

Fig. 1

5 (a) Derive, from first principles, the equation for the pressure drop in the fully-developed flow of an incompressible fluid through a cylindrical pipe of length L and diameter d, given that the wall shear stress is τ. [5]

(b) The 'Moody chart' in Fig. 2 shows the skin-friction coefficient for pipe flow as a function of Reynolds number and relative roughness.

(i) Define the physical variables ρ, V, d, ν and k, and explain why it is possible to summarise their influence in the form of this chart. [7]

(ii) Give a physical explanation for the region in which the skin-friction coefficient depends only on the relative roughness. [5]

(c) Methane gas with density 1.54 kg m^{-3} and kinematic viscosity 6.14×10^{-6} $m^2 s^{-1}$ flows through a 1 m diameter pipe at 10 m s^{-1}. The pipe material has roughness height $k = 0.05$ mm.

(i) Calculate the pressure drop over a 1 km length of pipe. (An accuracy of 5% is sufficient.) [2]

(ii) Discuss, quantitatively where possible, how the power required to pump the gas along the pipe would change if the diameter were altered while maintaining the same volumetric flow rate. (You can assume incompressible flow in the compressors used for the pumping.) [6]

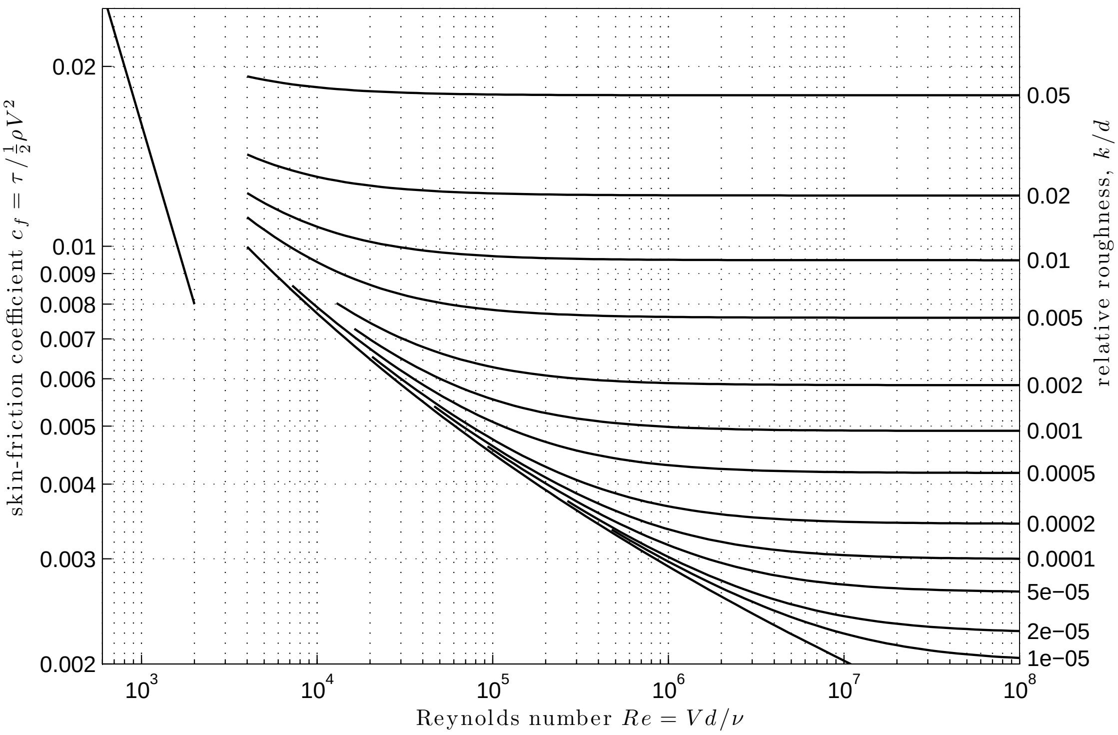
skin-friction coefficient $c_f = \tau / \frac{1}{2}\rho V^2$
0.02
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
10^3
10^4
10^5
10^6
10^7
10^8
Reynolds number $Re = Vd/\nu$
relative roughness, k/d
0.05
0.02
0.01
0.005
0.002
0.001
0.0005
0.0002
0.0001
5e−05
2e−05
1e−05

Fig. 2

6 (a) A large plenum contains air of density ρ at pressure p_p. The air exhausts through a gap, as shown in Fig. 3(a). Shortly after, at the plane A, the flow streamlines become parallel. Assuming that the flow can be treated as incompressible and inviscid,

(i) explain carefully why the air velocity V is uniform at A; [5]

(ii) find an expression for the plenum pressure relative to the atmospheric pressure, p_a. [2]

(b) The plenum of part (a) is supplied with air at volumetric flow rate Q via a pipe of area A, as shown in Fig. 3(b). The pipe flow is incompressible and uniform.

(i) What is the stagnation pressure of the pipe flow at exit? [4]

(ii) Compare your answer to part (i) with the plenum stagnation pressure, and state the reason for any difference. [4]

(c) A hovercraft of weight W is supported on an air cushion contained in a cylindrical plenum of diameter D. The air supply is provided through a pipe of area A by a compressor which draws from the surrounding atmosphere, and whose stagnation-pressure rise Δp_0 is linked to its flow rate Q by

$$\Delta p_0 = C_0 - C_2 Q^2,$$

where C_0 and C_2 are constants. Friction losses in the pipe are negligible.

(i) What is the plenum gauge pressure? [3]

(ii) Derive an expression for Q. [4]

(iii) Describe how you would use your results to obtain a lower-bound estimate for the gap between the skirt of the plenum and the ground. [3]

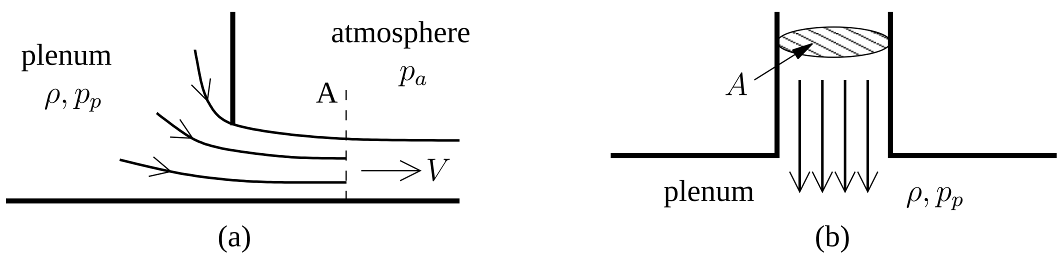

Fig. 3

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