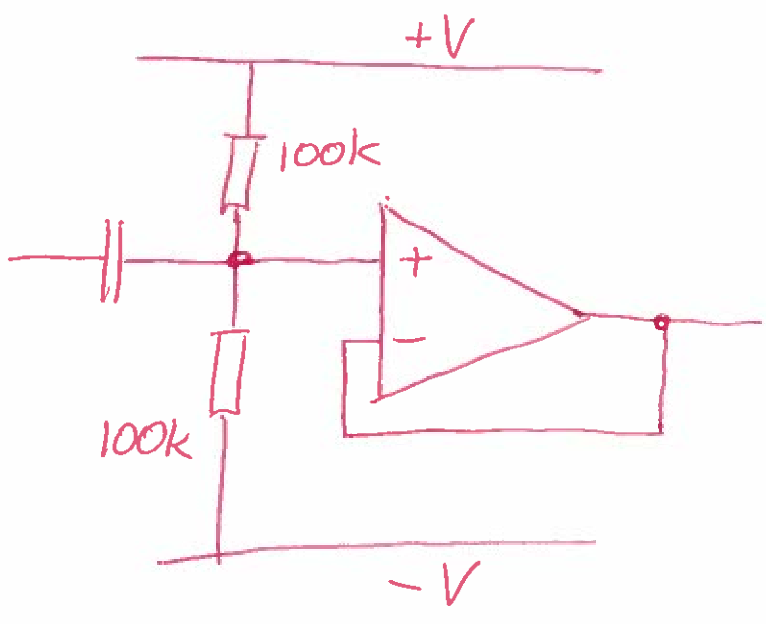
1B PAPER 5 2015
CRIB – DR. P. R. PALMER

1/ (a)(i) Finite bandwidth
Common-mode gain.
Finite output voltage
Finite input voltages.
Input bias currents
Input offset voltage.

(ii) There is no path for the dc bias current and nothing to set the dc voltage on the + input.

See 1A IEP:



1/(b)(i) Gain is $1 + \frac{50k}{R_G}$ (symmetry)

For 20 $R_G = \underline{\underline{2.63k}}$

For 1000 $R_G = \underline{\underline{50.05k}}$

(ii) Leave R_G open.

(iii) Instrumentation where common-mode noise is a problem, also high source impedance. eg. pH measurement in a chemical processing plant.

(c) Op-amp gain with frequency $\frac{A_1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$

With f.b. $\omega_{3dB} > \omega_c$

Gain-BW: $\frac{A_1}{\sqrt{1+\frac{(\omega_{3dB})^2}{(\omega_c)^2}}} \cdot \omega_{3dB} \simeq A_1 \omega_c$

The gain is set by the first stage (A_1, A_2)

$G \times BW$ $7 \times 1000 = 7000$, $70 \times 100 = 7000$

But $500 \times 10 = 5000$ so 2nd stage has cut in.

At $G = 10$ BW should be 700 kHz.

At 500 kHz first stage gives a gain of 14, 10/14

A_2 is in unity gain so $\underline{500\text{ kHz}}$ is 3dB of 2nd

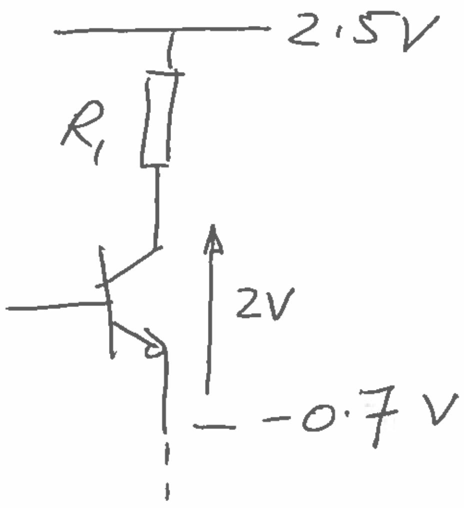
2 (a) CLASS A as Q_1 and Q_2 should not be cut off. (with Q_2 base to ground the input voltage should be close to ground)

(b) DC bias on the base-emitter of Q_1 in the on state $\doteq$ 0.6 – 0.8V

Current in R_3 = 2mA.

Voltage $-0.7-(-2.5)$ => $\underline{\underline{R_3 = 0.9k}}$

$V_{CE} = 2V$.

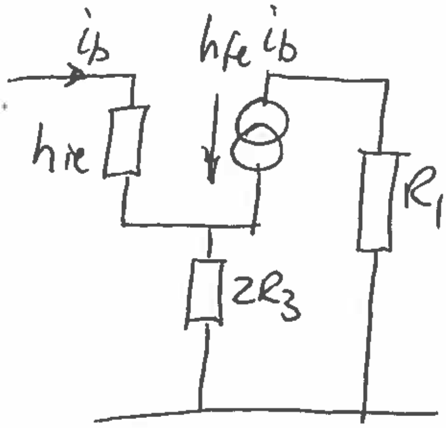


$\underline{\underline{R_1 = R_2 = 1.2k}}$

2 (c) Can do it in one go, or Differential Mode & Common Mode.

The second requires DM of $\frac{V_{in}}{2}$ & CM of the same.

CM.

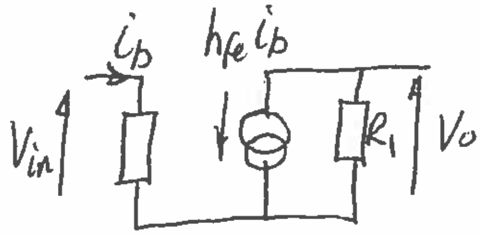


$$i_b = \frac{V_i - i_b(h_{fe} + 1)2R_3}{hie}, \quad V_o = -i_b h_{fe} R_1$$

$$i_b\, hie = V_i - (h_{fe} + 1) i_b . 2R_3$$

$$i_b = \frac{V_i}{hie + (h_{fe}+1)2.R_3} \quad \therefore \quad \frac{V_o}{V_i} = \frac{-h_{fe} R_1}{hie + (h_{fe}+1)2R_3}$$

DM



$$\frac{V_o}{V_i} = -\frac{h_{fe} R_1}{hie}.$$

Overall

$$\frac{V_o}{V_i} = -\frac{h_{fe} R_1}{2}\left[\frac{1}{hie} + \frac{1}{hie + (h_{fe}+1)2R_3}\right]$$

(d) DM $R_{IN} = h_{ie} = 1k$.

CM. $i_b = \dfrac{V_i - i_b(h_{fe}+1)2R_3}{h_{ie}}$.

$$i_b\left(h_{ie} + (h_{fe}+1)2R_3\right) = V_i.$$

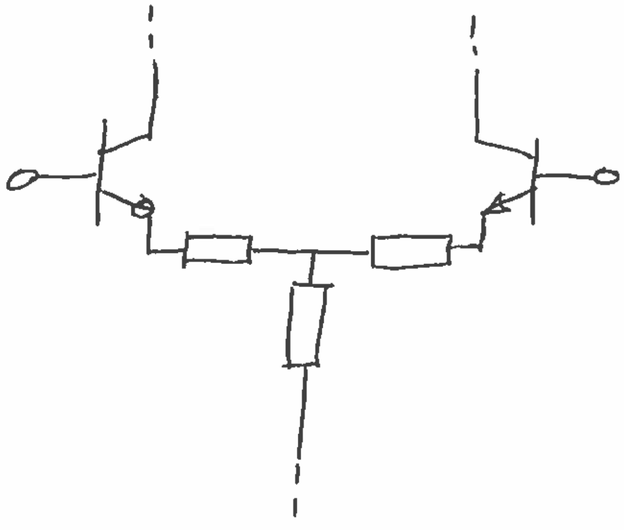
$$R_{IN} = h_{ie} + (h_{fe}+1)2R_3 = 1k + 201k \times 2$$

$$= 403k. \quad (R_3 \approx 1k)$$

So, as configured the input current is mostly DM ~ so $\frac{V_{IN}}{2} \cdot \frac{1}{h_{ie}}$ so $\underline{\underline{R_{IN} = 2h_{ie}}}$.

If drawn as a <u>whole</u> there are two h_{ie}'s to consider <u>in series</u>!

(e) The effect of R_3 is excellent $(h_{fe}+1)R_3$ so a small resistor in the emitters of Q_1 & Q_2



3 a/ ~~ψ~~ Loads expect balanced voltages

2/ The currents sum to zero at the neutral ~ often no neutral used.

3/ Nice fluxes in machines/generators.

(b) The power factor deals with power ($\int$ energy dt) and reactive power ($\int$ energy dt)

Which are not phasor quantities in time.

Therefore the load is a "black box".

$$P = \sqrt{3}\, V_L I_L \cos\phi$$

Really there are 3 ψ loads, but either branch

$V_L = \sqrt{3} V_p$ <u>OR</u> $I_L = \sqrt{3} I_p$.

$$\text{eg} \quad P = \sqrt{3}\, V_L \sqrt{3} I_{ph} \cos\phi$$

$$= 3\, V_L I_{ph} \cos\phi$$

The 3 branch loads are there still!

c)(i) 33kV 12.000 MW. 0.8pF

$$Q = \frac{12}{0.8} \sin\cos^{-1}0.8 = 9\,MVAR.$$

$$\frac{Q_T}{3} = 3\,MVAR = \left(\frac{33k}{\sqrt{3}}\right)^2 . \omega C \qquad \underline{\underline{C = 26\mu F}}$$

(ii) $$I_L = \frac{12\cancel{M}k}{\sqrt{3}.33\cancel{k}.} = 210A.$$

$$I^2R = 0.132\,MW.$$

$$I^2\omega L = 210^2 . 314 \times 36m = 0.499\,MVAr.$$

$$P_{TOT} = 12 + 3\times 0.132 \quad MW$$

$$Q_{TOT} = 0.499\times 3 \quad MVAR, \; S_{TOT} = \sqrt{P_{TOT}^2 + Q_{TOT}^2}$$

$$= 12.486$$

$$V_{Lg} = \frac{12.486M}{210\sqrt{3}} = \underline{\underline{34.3kV}}$$

d/(i) Extra 5 μF at the load in Delta.

$$Q_C = 3 \times (33k)^2 \times 314 \times 5\mu = 5.13 \text{ MVAR}$$

$$S_T = \sqrt{12^2 + 5.13^2} \simeq 13$$

$$I = \frac{13}{\sqrt{3} \times 33k} = 227A$$

$$S_{TOT} = \sqrt{(12 + 3 \times 227^2 \times 3)^2 + 5^2} = 13.43$$

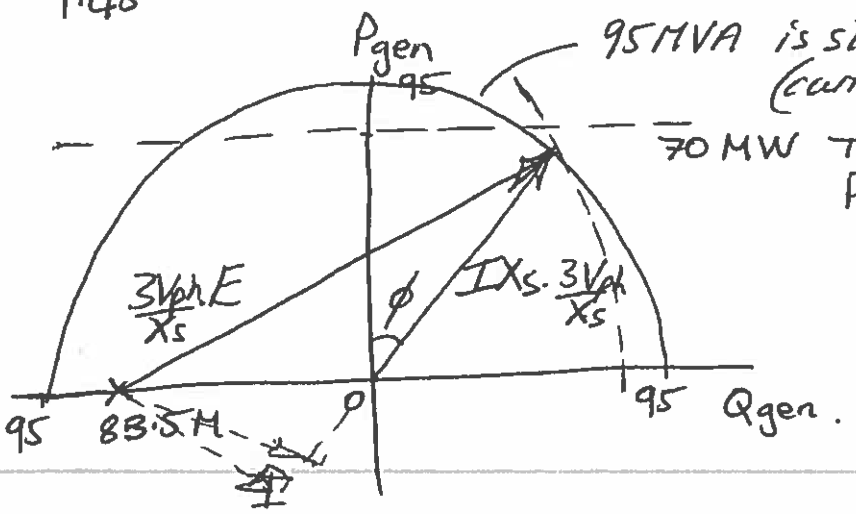
$$V_{Lg} = \frac{13.43}{227\sqrt{3}} = 34.2 \text{ kV}.$$

(ii) 50 km. ⇒ 25 μF at the load.
Could remove the 26 μF but
maybe distributed.

Also 25 μF at the generator so
high currents at that end.

4 (a) Load chart uses $\frac{3V_{ph}^2}{X_s}$ on x axis.

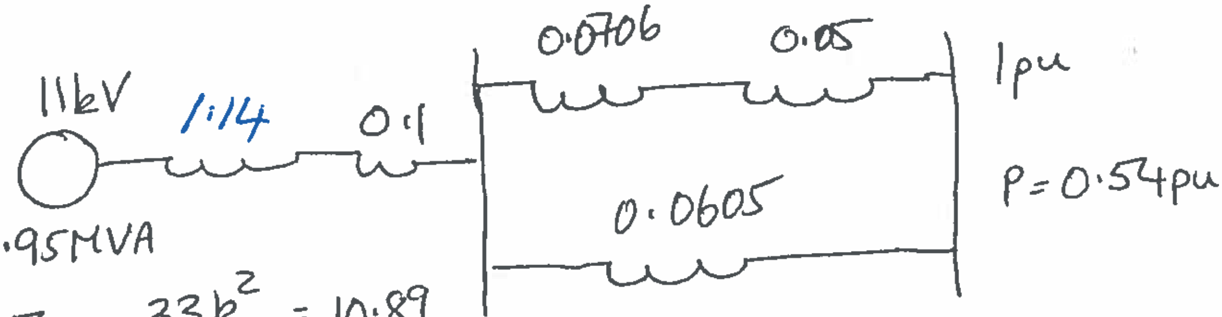
$$\frac{3 \times (11kV/\sqrt{3})^2}{1.45} = 83.5 \text{ MVAR}.$$



0.85 p.f. $\cos^{-1} 0.85 = 32° = \phi$ Sets max E (rotor heating)

ie. 95MVA at a p.f. of 0.85

b/



$Z_b = \frac{33k^2}{100M} = 10.89$

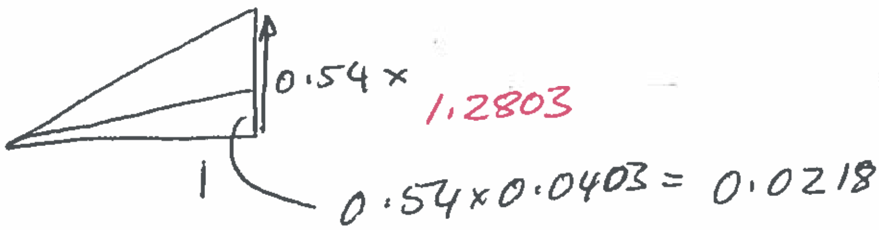
Line 1 $0.7686j\Omega \rightarrow 0.0706$

Line 2 $0.6588j\Omega \rightarrow 0.0605$

c/ Lines 1&2 in parallel now.

$$\frac{0.0605 \times 0.1206}{0.1811} = 0.0403$$

$I = 0.54$



$$I_{line\,2} = \frac{0.0218}{0.0605} = 0.3597$$

$$I_b = \frac{100M}{\sqrt{3} \cdot 33k} = 1{,}749.6$$

$I_{line\,2} = 629.3A$

$\therefore I_{line\,1} = 315.7A$

Note dropped 'j's as all reactive

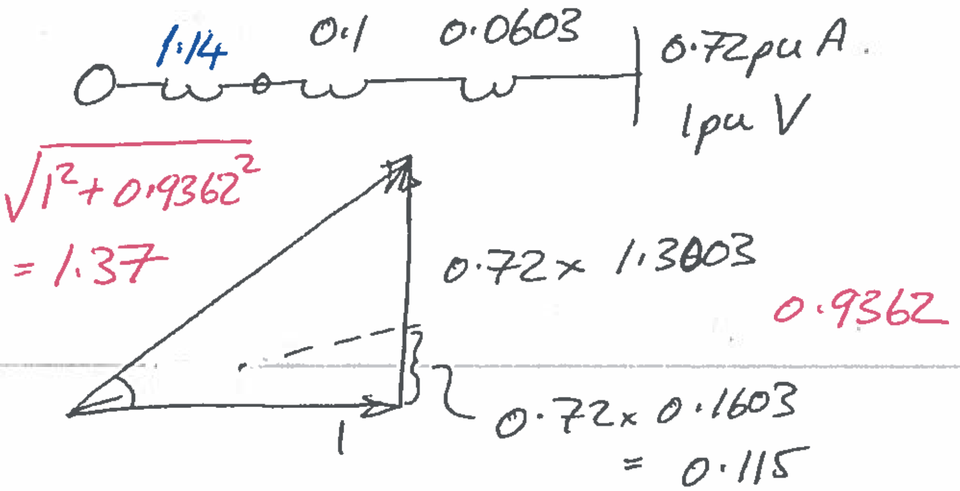
d/ (i) Need to Add 0.0601 pu ie 0.654 jΩ

(ii) 2 lines with 0.1206 pu. ⇒ 0.0603 pu

∴ current in each line is 0.3597 pu.

⇒ 0.7194 pu. total ⇒ 72 MW

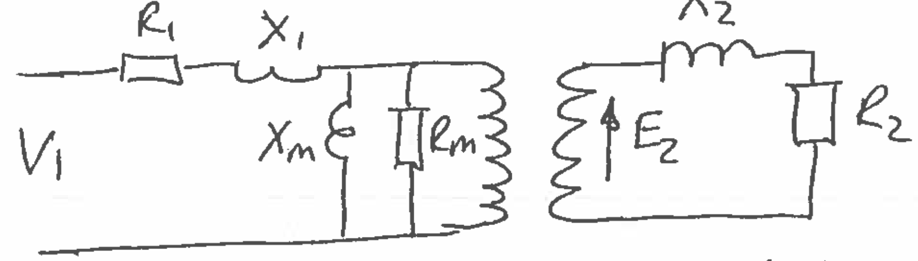
(iii)



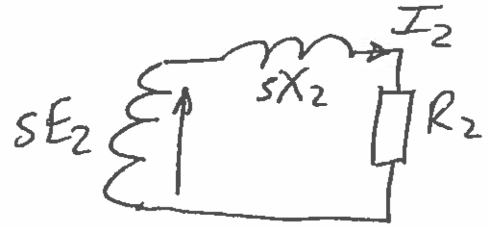
$\tan^{-1} 0.9362 - \tan^{-1} 0.115 = \underline{\underline{36.5^\circ}}$

5(a) Slip is the fractional difference in speed between the rotor and the magnetic field in the airgap set by the stator. Slip is necessary to provide a voltage to the rotor circuit, whereby a current flows and torque is produced.

(b)



as a transformer. When rotating sE_2



$$I_2 = \frac{sE_2}{R_2 + jX_2 \cdot s} = \frac{E_2}{\frac{R_2}{s} + jX_2}$$

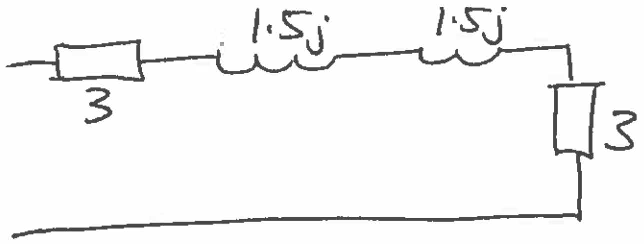
Next use power

$$P_{gap} = E_2 I_2 \cos\phi = E_2 \cdot \frac{E_2}{\sqrt{(\frac{R_2}{s})^2 + (X_2)^2}} \cdot \frac{R_2/s}{\sqrt{(\frac{R_2}{s})^2 + (X_2)^2}}$$

$$T\omega_s = \frac{E_2^2 \, R_2/s}{(\frac{R_2}{s})^2 + X_2^2}$$

$\frac{R_2}{s}$ is mechanical power + losses.

(C) (i)



$$I_2' = \frac{415/\sqrt{3}}{6+3j} \qquad I_2' = 35.7A$$

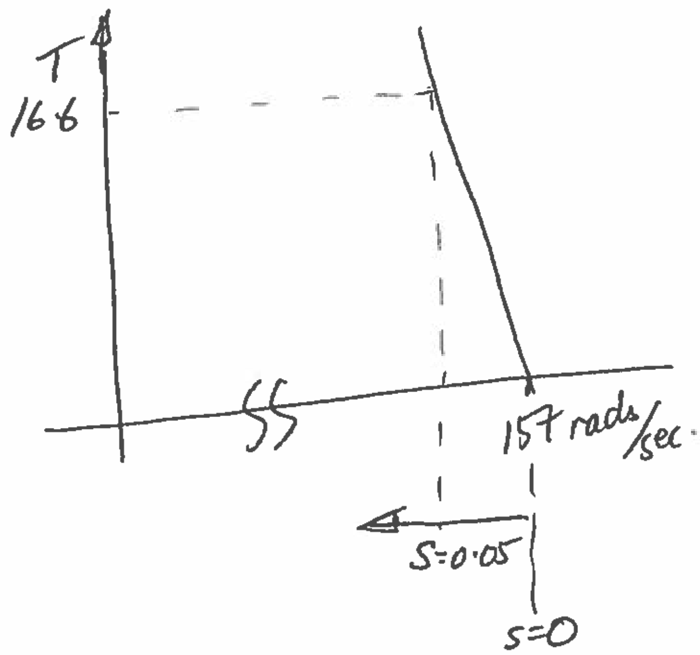
$$T = \frac{3 I_2'^2}{\omega_s} \cdot \frac{R_2}{s} = \frac{3 \times 35.7^2 \times 3}{50\pi} = \underline{\underline{73\,Nm}}$$

(ii) $s = \frac{1500-1425}{1500} = 0.05$

$$I_2' = \frac{415/\sqrt{3}}{3 + \frac{3}{0.05} + 3j} = 3.81A$$

$$T = \frac{3 \times 3.81^2}{157} \cdot \frac{3}{0.05} = \underline{\underline{16.6\,Nm.}}$$

(iii)



(d) Using the graph as a straight line

$s = 0.05 \times 0.80 = 0.04$.

$T = 16.6 \times 0.80$

$s = 0.04 \Rightarrow 1500\,rpm - 60\,rpm = \underline{\underline{1440\,rpm}}$

(The value of $R_2/s = 3/0.05 = 60\Omega$.

$60\Omega \gg (3+3j)$

and R_2/s gets bigger as $s \to 0$!)

6 (a) Databook. (Free space)

$$E_x = E_0 e^{j(\omega t - \beta z)}$$
$$H_y = H_0 e^{j(\omega t - \beta z)}$$

travelling waves in z direction.

Maxwell $\nabla \times H = \dot{D} = \varepsilon_0 \dot{E} = \varepsilon_0 \frac{dE}{dt}$

or $\left(\nabla \times E = -\dot{B} = -\frac{dB}{dt} \right)$

$$\frac{dH_y}{dz} = \varepsilon \frac{dE_x}{dt}$$

$$\therefore H_0 . -\beta \cancel{e^{j(\omega t - \beta z)}} = \varepsilon E_0 \omega \cancel{e^{j(\omega t - \beta z)}} \quad \text{— (1)}$$

$$\frac{H_0}{E_0} = -\frac{\varepsilon \omega}{\beta} \qquad \beta = \omega \sqrt{\mu \varepsilon}$$

$$= -\frac{\varepsilon}{\sqrt{\mu \varepsilon}} = -\sqrt{\frac{\varepsilon}{\mu}} = -\frac{1}{Z_0}$$

Looking at (1) H_0 & E_0 are maximum at the same z & t, and travelling at v=c. Both represent ~~[illegible]~~ stresses, so <u>energy</u> flows in the direction of z.

b (i) Need the travel time around trip.

Shortest time at 100 km.

$$t = \frac{200\times10^3}{3\times10^8} = \frac{2}{3}\,\mathrm{ms} = \underline{\underline{0.667\,\mu s}}$$

(ii) Databook $\dfrac{\eta_2-\eta_1}{\eta_2+\eta_1} = \dfrac{\sqrt{\frac{\mu_0}{1.5\varepsilon_0}}-\sqrt{\frac{\mu_0}{\varepsilon_0}}}{\sqrt{\frac{\mu_0}{1.5\varepsilon_0}}+\sqrt{\frac{\mu_0}{\varepsilon_0}}} = \dfrac{\frac{1}{\sqrt{1.5}}-1}{\frac{1}{\sqrt{1.5}}+1}$

$$= \frac{0.18\cancel{4}}{1.816}$$

$$= -0.10$$

Isotropic radiation $\times G$, distance r

$$P_t = \frac{850\times10^3}{4\pi r^2}\times G \qquad P_t\times 0.1^2 \text{ reflected at cloud.}$$

i.e. for $1\,\mathrm{W\,m^{-2}}$, $P_t = 100\,\mathrm{W\,m^{-2}}$

$$G = \frac{4\pi\times25\times10^{10}}{850\times10^3}\times100 = \frac{\pi}{85}\times10^{10}$$

$$= 0.37\times10^9$$

This is huge, but $100\,\mathrm{W/m^2}$ at 500km is huge!

(c) For $1\,m^2$ of cloud....

$G = 500$, $1\,W$, $4\,m^2$ antenna

$$P_r = \frac{1 \times 500 \times 4}{4\pi \times 25 \times 10^{10}}$$

$$= \frac{20}{\pi} \times 10^{-10}\,W = \frac{1}{2}\hat{I}^2 Z_0$$

$$\hat{I}^2 = \frac{40}{75\pi} \times 10^{-10}$$

$$\underline{\underline{\hat{I} = 4.12 \times 10^{-6}\,A}}$$

(d) at 90 GHz $\quad \lambda = \frac{3 \times 10^8}{90 \times 10^9} = \frac{1}{300}\,m.$

This is a nice size for raindrops so they will act like cats eyes

(Also little atmospheric loss)

7 (a)(i) $v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_r . \mu_0 \mu_r}}$

In a free space system $v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$, so the same eq'n but no materials.

The waves travel $A(t,x) = A_0 \cos(kx - \omega t)$

eg. $V(x,t) = V_F e^{j(\omega t - \beta x)}$ (<u>infinite line</u>)

Its a point of constant phase which moves: ie $\omega t = \beta x$ $\therefore \frac{x}{t} = \frac{\omega}{\beta} = v$

v is materials based so constant so $\beta \propto \omega$.

(ii) $Z_0 = \sqrt{\frac{L}{C}}$ $Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}} \ln(r_2/r_1)$ $\eta = \sqrt{\frac{\mu}{\varepsilon}}$

The coax constrains the fields, whereas the y & x directions are unlimited in free space.

(iii) $v = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_r \mu_0 \mu_r}}$ ~ light in space $\frac{1}{\sqrt{\varepsilon_0 \mu_0}}$

so $\frac{1}{\mu_r \varepsilon_r}$ is different by $(0.66)^2$. The insulators usually have $\mu_r = \mu_0$ so ε_r is 2.3.

(a) (iii) cont.

from (ii) $Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ \ln(r_2/r_1) = 50$

$$\frac{1}{2\pi}\sqrt{\frac{1.257\times10^{-6}}{2.3\times 8.85\times10^{-12}}}\cdot \ln r_2/r_1 = 50$$

$$\ln r_2/r_1 = \frac{100\pi}{0.25\times10^{3}} = \pi/2.5$$

eg. if $r_1 = 1\text{mm} \Rightarrow r_2 = 3.5\text{mm}$.

(b)(i) $\rho_L = \frac{50-50}{100} = 0 \Rightarrow$ no reflections.

The 50Ω cable has a "characteristic" impedance, but the 50Ω source & 50Ω load have real impedances, so form a potential divider. $\Rightarrow \underline{\underline{E = 2V}}$

(ii) Matched at source so no repeated reflections.

$\rho_L = 1$ Delay $\frac{1\text{m}}{0.66\times 3\times10^{8}} = 5\text{ns}$

1V at source $\Rightarrow$ 1V out.

(0.5V into the line, 100% reflection ∴ 1V out)

$\underline{\underline{1V}}$

(b)(ii) Fully matched 2V at DC.

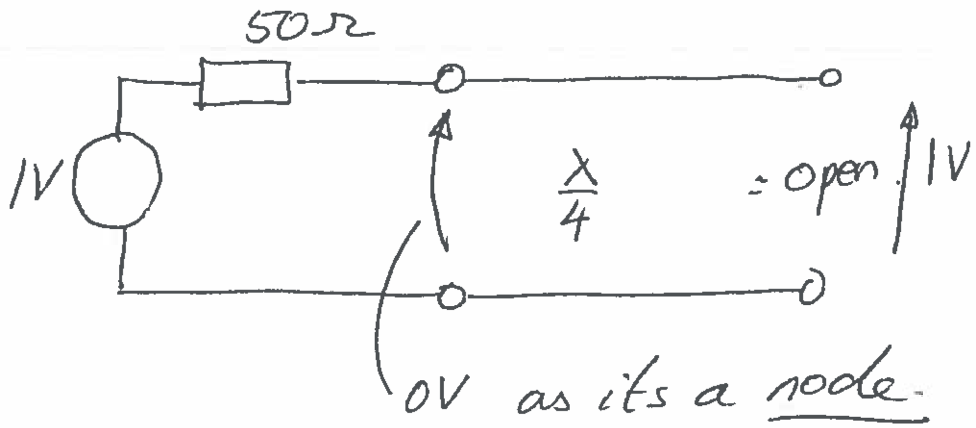
(2V at pulse as T.L. "disappears")

$P = 2^2/100 = 40\,mW$.

Unmatched $P = 1^2/1\times10^6 = \underline{1\,\mu W}$.

(c). 50 MHz $\frac{1}{50\times10^6} = 20\,ns$ period.

5ns delay so 1m is $\frac{\lambda}{4}$



(Power = $\frac{1^2}{50}$ = 20mW.)