EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 19 April 2016 2 to 3.30

Module 4A15

AEROACOUSTICS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
Engineering Data Book
CUED approved calculator allowed
Attachment: 4A15 Aeroacoustics data sheet (6 pages).

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Small explosive charges are often used as omnidirectional sound sources in underwater acoustics. The waveform of the pressure pulse produced is an abrupt rise followed by a tail that resembles a decaying exponential function. A charge is detonated at time $t = 0$ and the following data are measured in the free field at a distance r_0 from the charge:

$$p(r_0,t) = \begin{cases} 0, & t < r_0/c_0 \\ A\left(1 - \frac{t - r_0/c_0}{t_0}\right) e^{-(t - r_0/c_0)/t_0}, & t > r_0/c_0 \end{cases}$$

(a) Find the source strength $dQ(t)/dt$ by assuming the charge to be a point source. Q is the mass injection rate per unit volume. [60%]

(b) Obtain the expression for the pressure at any distance r in an infinite ocean. [10%]

(c) Obtain the total energy radiated to the far field. [30%]

Hint:

$$\int_0^\infty (t_0 - t)^2 e^{-2t/t_0} \mathrm{d}t = \frac{t_0^3}{4}$$

2 Consider a low Mach-number, isentropic, cold, turbulent jet.

(a) By finding the sound radiated from a single compact eddy, show that the acoustic far-field density from the eddy scales as

$$\rho' \sim \rho_0 \frac{x_i}{x}\frac{x_j}{x}\frac{l}{x}m^4$$

where ρ_0 is the ambient density, x is the distance from the source to the observer, x_i represents the component of $\mathbf{x}$ in the i-direction, l is the length scale of the eddy, $m = u'/c_0$, u' is the velocity scale of the eddy and c_0 is the speed of sound. [60%]

Hint: $$\frac{\partial^2}{\partial x_i \partial x_j}\int T_{ij}\left(\mathbf{y}, t - \frac{x}{c_0}\right)\mathrm{d}\mathbf{y} = \frac{1}{c_0^2}\left(\frac{x_i x_j}{x^2}\right)\frac{\partial^2}{\partial t^2}\int T_{ij}\left(\mathbf{y}, t - \frac{x}{c_0}\right)\mathrm{d}\mathbf{y}$$

(b) Use this result to deduce an expression for the acoustic power radiated by the jet (Lighthill's eighth power law). [40%]

3 An acoustic liner for low frequency sound absorption consists of a perforated surface with an open-area ratio of 5%. Each hole opens onto a cavity of volume $0.0001\,\mathrm{m}^3$ and together the hole and cavity behave like a Helmholtz resonator. The difference between the pressure perturbation just outside a hole and within its cavity is equal to

$$\rho_0 l \frac{\partial u'}{\partial t} + \alpha u'$$

where ρ_0 is the mean density, l is 0.6 times the hole diameter, u' is the velocity of the air flowing through the hole, $\alpha = \rho_0 c_0 k$ where k is a positive constant and c_0 is the speed of sound. The ambient temperature is 600 K and pressure 1 bar.

(a) For sound of frequency ω, determine the relationship between the pressure $p_1'(t)$ at a hole and u', the velocity of air through it in terms of ω, k and the hole diameter d. [30%]

(b) What hole diameter d would you choose to have resonance at 400 Hz for $k = 0$? [20%]

(c) With this choice of hole size for sound at 400 Hz and with $k = 0.1$:

(i) determine the rate of absorption of sound energy by a single hole in terms of the mean square pressure perturbation at the hole. [30%]

(ii) determine the rate of absorption of sound energy per unit area of the liner in terms of the mean square pressure perturbation at the liner surface. [20%]

4 (a) A thin wall of mass per unit area m is located in the plane $x = 0$ and is free to make small oscillations in the x direction. The wall separates two acoustic media, of sound speed and density c_0 and ρ_0 in $x < 0$ and c_1 and ρ_1 in $x > 0$. A plane sound wave is normally incident on the wall from $x < 0$.

(i) Determine an expression for the ratio of the magnitudes of the reflected and incident pressures. [40%]

(ii) Identify two dimensionless numbers which determine the size of this quantity, and explain their physical significance. [10%]

(b) High frequency sound propagates through a continuous medium whose sound speed, $c(x)$, is given by

$$c(x) = c_0 \exp(x/l) \ , \qquad (1)$$

where c_0 and l are the reference speed of sound and length, respectively. Sound is emitted from a source located at $x = y = 0$ at an angle θ_0 to the x-axis.

(i) Using ray theory, find the equation of the ray. (You may assume without proof that

$$\int \frac{\alpha \exp(x/l)}{\sqrt{1-\alpha^2 \exp(2x/l)}} \mathrm{d}x = l \sin^{-1}(\alpha \exp(x/l)) + k \ , \qquad (2)$$

where k is a constant.) [25%]

(ii) Sketch a graph showing the rays corresponding to $\theta_0 = \pi/4$ and $\theta_0 = \pi/2$. [15%]

(iii) Without making any detailed calculations, explain briefly how one determines the variation of the acoustic pressure along a ray. [10%]

END OF PAPER

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